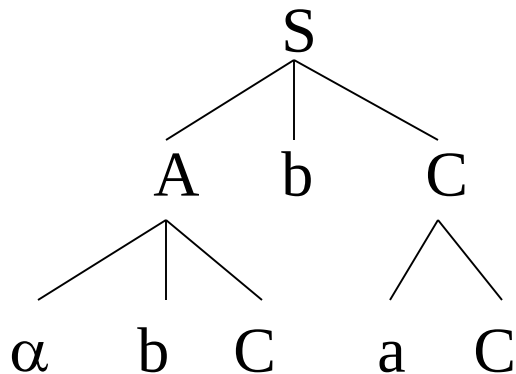


Chapter 6 LR Parsing Techniques

Shift-Reduce Parsers

- Reviewing some technologies:
 - Phase
 - Simple
 - Handle of a sentential form



A sentential form

$\alpha \ b \ C \ b \ a \ C$
 $\underbrace{\hspace{1.5cm}}_{\text{handle}} \quad \underbrace{\hspace{1.5cm}}_{\text{Simple phase}}$

Shift-reduce parser

- A parse stack
 - Initially empty, contains symbols already parsed
 - Elements in the stack are not terminal or nonterminal symbols
 - The parse stack catenated with the remaining input always represents a right sentential form
 - Tokens are shifted onto the stack until the top of the stack contains the handle of the sentential form

Shift-reduce parser

- Two questions
 1. Have we reached the end of handles and how long is the handle?
 2. Which nonterminal does the handle reduce to?
- We use tables to answer the questions
 - ACTION table
 - GOTO table

Shift-reduce parser

- LR parsers are driven by two tables:
 - Action table, which specifies that actions to take
 - Shift, reduce, accept or error
 - Goto table, which specifies state transition
- We push states, rather than symbols onto the stack
- Each state represents the possible subtrees of the parse tree

Shift-reduce parser

1. $\langle \text{program} \rangle \rightarrow \mathbf{begin} \langle \text{stmts} \rangle \mathbf{end} \$$
2. $\langle \text{stmts} \rangle \rightarrow \text{SimpleStmt} ; \langle \text{stmts} \rangle$
3. $\langle \text{stmts} \rangle \rightarrow \mathbf{begin} \langle \text{stmts} \rangle \mathbf{end} ; \langle \text{stmts} \rangle$
4. $\langle \text{stmts} \rangle \rightarrow \lambda$

Symbol	State											
	0	1	2	3	4	5	6	7	8	9	10	11
begin	S	S			S		S			S		
end		R4	S		R4		R4	S		R4	R2	R3
;						S			S			
SimpleStmt		S			S		S			S		
\$				A								
$\langle \text{program} \rangle$												
$\langle \text{stmts} \rangle$		S			S		S			S		

Figure 6.2 A Shift-Reduce **action** Table for G_0

Symbol	State											
	0	1	2	3	4	5	6	7	8	9	10	11
begin	1	4			4		4			4		
end			3					8				
;						6			9			
SimpleStmt		5			5		5			6		
\$												
$\langle \text{program} \rangle$												
$\langle \text{stmts} \rangle$		2			7		10			11		

Figure 6.3 A Shift-Reduce **go_to** Table for G_0

```

void shift_reduce_driver(void)
{
    /*
     * Push the Start State, S0,
     * onto an empty parse stack.
     */
    push(S0);

    while (TRUE) {    /* forever */
        /*
         * Let S be the top parse stack state;
         * let T be the current input token.
         */

        switch (action[S][T]) {
        case ERROR:
            announce_syntax_error();
            break;

        case ACCEPT:
            /* The input has been correctly parsed. */
            clean_up_and_finish();
            return;

        case SHIFT:
            push(go_to[S][T]);
            scanner(& T);    /* Get next token. */
            break;

        case Reducei:
            /*
             * Assume i-th production is  $X \rightarrow Y_1 \cdot \cdot \cdot Y_m$ .
             * Remove states corresponding to
             * the RHS of the production.
             */
            pop(m);
            /* S' is the new stack top. */
            push(go_to[S'][X]);
            break;

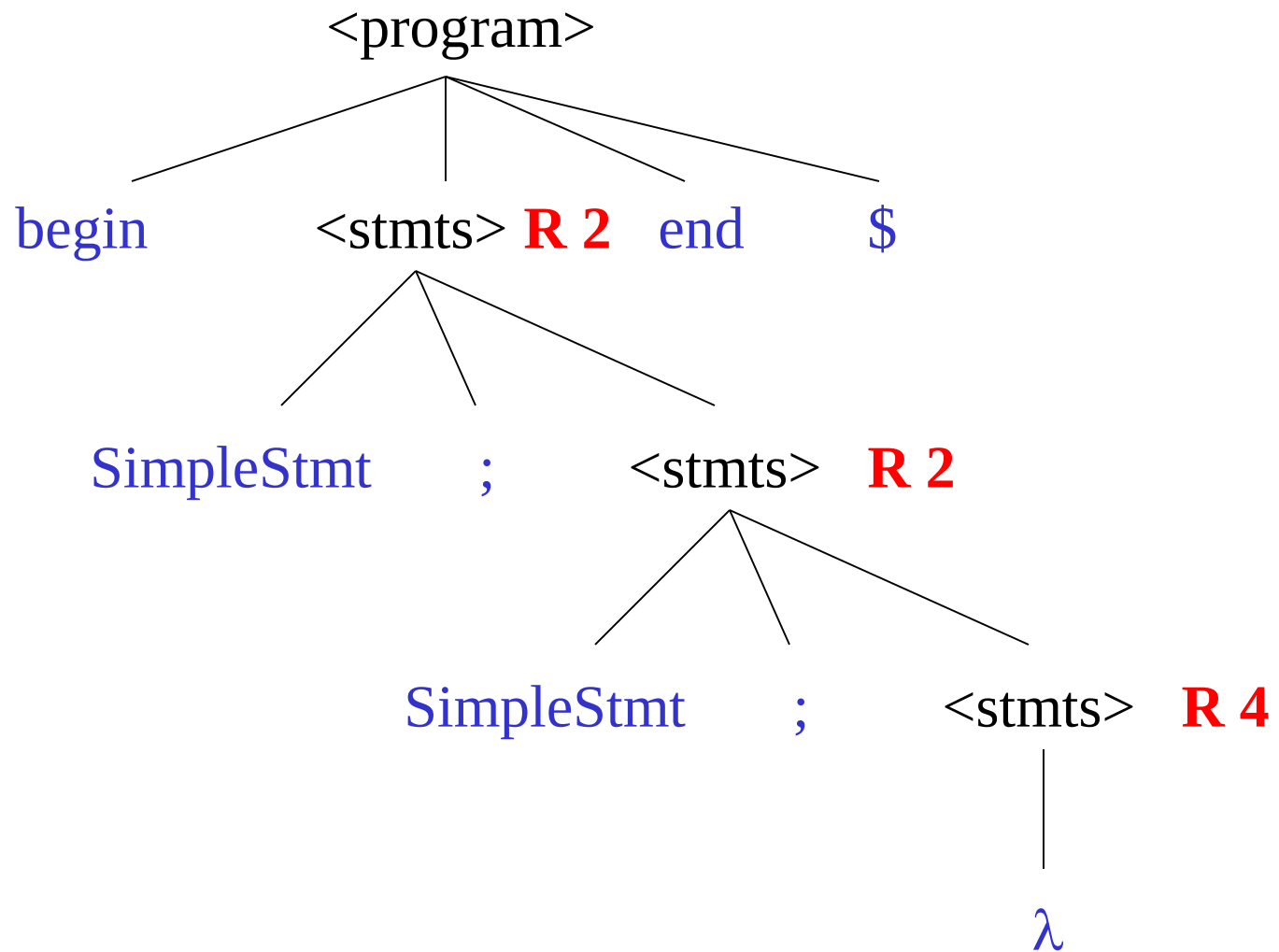
        }
    }
}

```

Figure 6.1 A Simple Shift-Reduce Driver

Step	Parse Stack	Remaining Input	Parser Action
(1)	0	begin SimpleStmt ; SimpleStmt ; end \$	Shift
(2)	0,1	SimpleStmt ; SimpleStmt ; end \$	Shift
(3)	0,1,5	; SimpleStmt ; end \$	Shift
(4)	0,1,5,6	SimpleStmt ; end \$	Shift
(5)	0,1,5,6,5	; end \$	Shift
(6)	0,1,5,6,5,6	end \$	Reduce 4
(7)	0,1,5,6,5,6,10	end \$	Reduce 2
(8)	0,1,5,6,10	end \$	Reduce 2
(9)	0,1,2	end \$	Shift
(10)	0,1,2,3	\$	Accept

Figure 6.4 Example of a Shift-Reduce Parse



LR Parsers

- LR(1):
 - left-to-right scanning
 - rightmost derivation(reverse)
 - 1-token lookahead
- LR parsers are deterministic
 - no backup or retry parsing actions
- LR(k) parsers
 - decide the next action by examining the tokens already shifted and at most k lookahead tokens
 - the most powerful of deterministic bottom-up parsers with at most k lookahead tokens.

LR(0) Parsing

- A production has the form
 - $A \rightarrow X_1 X_2 \dots X_j$
- By adding a dot, we get a configuration (or an item)
 - $A \rightarrow \bullet X_1 X_2 \dots X_j$
 - $A \rightarrow X_1 X_2 \dots X_i \bullet X_{i+1} \dots X_j$
 - $A \rightarrow X_1 X_2 \dots X_j \bullet$
- The $\bullet$ indicates how much of a RHS has been shifted into the stack.

LR(0) Parsing

- An item with the • at the end of the RHS
 - $A \rightarrow X_1 X_2 \dots X_j \bullet$
 - indicates (or recognized) that RHS should be reduced to LHS
- An item with the • at the beginning of RHS
 - $A \rightarrow \bullet X_1 X_2 \dots X_j$
 - predicts that RHS will be shifted into the stack

LR(0) Parsing

- An LR(0) state is a set of configurations
 - This means that the actual state of LR(0) parsers is denoted by one of the items.
- The closure0 operation:
 - if there is an configuration $\mathbf{B} \rightarrow \delta \cdot \mathbf{A} \rho$ in the set then add all configurations of the form $\mathbf{A} \rightarrow \cdot \gamma$ to the set.
- The initial configuration
 - $s_0 = \text{closure0}(\{\mathbf{S} \rightarrow \cdot \alpha \$\})$

LR(0) Parsing

```
configuration_set closure0(configuration_set s)
{
    configuration_set s' = s;

    do {
        if (B  $\rightarrow$   $\delta \bullet A\rho \in s'$  for  $A \in V_n$ ) {
            /*
             * Predict productions with A
             * as the left-hand side.
             */
            Add all configurations of the form
                A  $\rightarrow \bullet \gamma$  to s'
        }
    } while (more new configurations can be added)

    return s';
}
```

Figure 6.5 An Algorithm to Close LR(0) Configuration Sets

$S \rightarrow E\$$

$E \rightarrow E + T \mid T$

$T \rightarrow ID \mid (E)$

$\text{closure}_0(\{S \rightarrow \bullet E\$ \}) = \{$

- $S \rightarrow \bullet E\$,$
- $E \rightarrow \bullet E + T,$
- $E \rightarrow \bullet T,$
- $T \rightarrow \bullet ID,$
- $T \rightarrow \bullet (E) \}$

LR(0) Parsing

- Given a *configuration set* **s**, we can compute its successor, **s'**, under a symbol **X**
 - Denoted $\text{go_to0}(s, X) = s'$

```
configuration_set go_to0(configuration_set s, symbol X)
{
    sb = ∅;
    for (each configuration c ∈ s)
        if (c is of the form A → β • X γ)
            Add A → βX • γ to sb;

    /*
     * That is, we advance the • past the symbol X,
     * if possible. Configurations not having a
     * dot preceding an X are not included in sb.
     */

    /* Add new predictions to sb via closure0. */
    return closure0(sb);
}
```

Figure 6.6 An Algorithm to Compute the LR(0) **go_to** Function

LR(0) Parsing

- Characteristic finite state machine (CFSM)
 - It is a finite automaton, p.148, para. 2.
 - Identifying configuration sets and successor operation with CFSM states and transitions

```
void build_CFSM(void)
{
    Create the Start State of the CFSM; Label it with  $s_0$ 
    Create an Error State in the CFSM; Label it with  $\emptyset$ 

    S = SET_OF(  $s_0$  );

    while(S is nonempty) {
        Remove a configuration set s from S;
        /* Consider both terminals and nonterminals */
        for (X in Symbols) {
            if (go_to0(s,X) does not label a CFSM state) {
                Create a new CFSM state and label it
                    with go_to0(s,X);
                Put go_to0(s,X) into S;
            }
            Create a transition under X from the state s
                labels to the state go_to0(s,X) labels;
        }
    }
}
```

Figure 6.7 An Algorithm to Compute the CFSM for a Grammar

LR(0) Parsing

- For example, given grammar G_2

$S' \rightarrow S\$$

$S \rightarrow ID|\lambda$

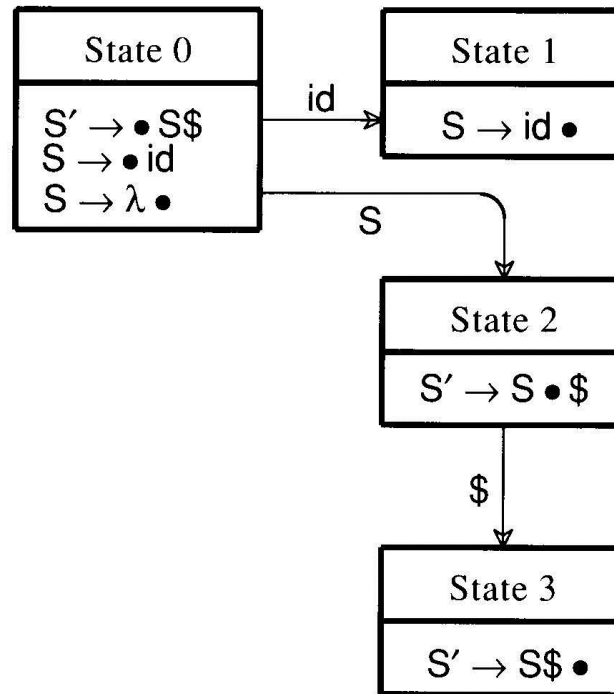


Figure 6.8 CFSM for G_2

LR(0) Parsing

- CFSM is the goto table of LR(0) parsers.

```
int ** build_go_to_table(finite_automaton CFSM)
{
    const int N = num_states(CFSM);
    int **tab;

    Dynamically allocate a table of dimension
    N × num_symbols(CFSM) to represent
    the go_to table and assign it to tab;

    Number the states of CFSM from 0 to N-1,
    with the Start State labeled 0;

    for (S = 0; S <= N - 1; S++) {
        /* Consider both terminals and nonterminals. */
        for (X in Symbols) {
            if (State S has a transition under X
                to some state T)
                tab[S][X] = T;
            else
                tab[S][X] = EMPTY;
        }
    }
    return tab;
}
```

Figure 6.9 An Algorithm to Build the LR(0) go_to Table

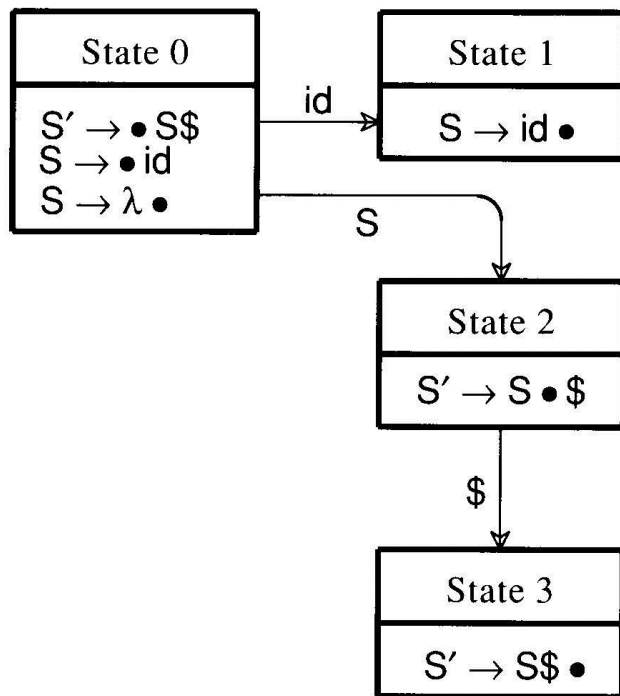


Figure 6.8 CFSM for G_2

State	Symbol		
	ID	\$	S
0	1	4	2
1	4	4	4
2	4	3	4
3	4	4	4
4			

Figure 6.10 A **go_to** Table for Grammar G_2

LR(0) Parsing

- Because LR(0) uses no lookahead, we must extract the **action** function directly from the configuration sets of **CFSM**
- Let $Q = \{\text{Shift}, \text{Reduce}_1, \text{Reduce}_2, \dots, \text{Reduce}_n\}$
 - There are **n** productions in the CFG
- S_0 be the set of CFSM states
 - $P: S_0 \rightarrow 2^Q$
- $P(s) = \{\text{Reduce}_i \mid B \rightarrow \rho \bullet \in s \text{ and production } i \text{ is } B \rightarrow \rho\} \cup (\text{if } A \rightarrow \alpha \bullet a\beta \in s \text{ for } a \in V_t \text{ Then } \{\text{Shift}\} \text{ Else } \emptyset)$

LR(0) Parsing

- G is LR(0) if and only if $\forall s \in S_0 \ |P(s)|=1$
- If G is LR(0), the action table is trivially extracted from P
 - $P(s)=\{\text{Shift}\} \Rightarrow \text{action}[s]=\text{Shift}$
 - $P(s)=\{\text{Reduce}_j\}$, where production j is the augmenting production, $\Rightarrow \text{action}[s]=\text{Accept}$
 - $P(s)=\{\text{Reduce}_i\}$, $i \neq j$, $\text{action}[s]=\text{Reduce}_i$
 - $P(s)=\emptyset \Rightarrow \text{action}[s]=\text{Error}$

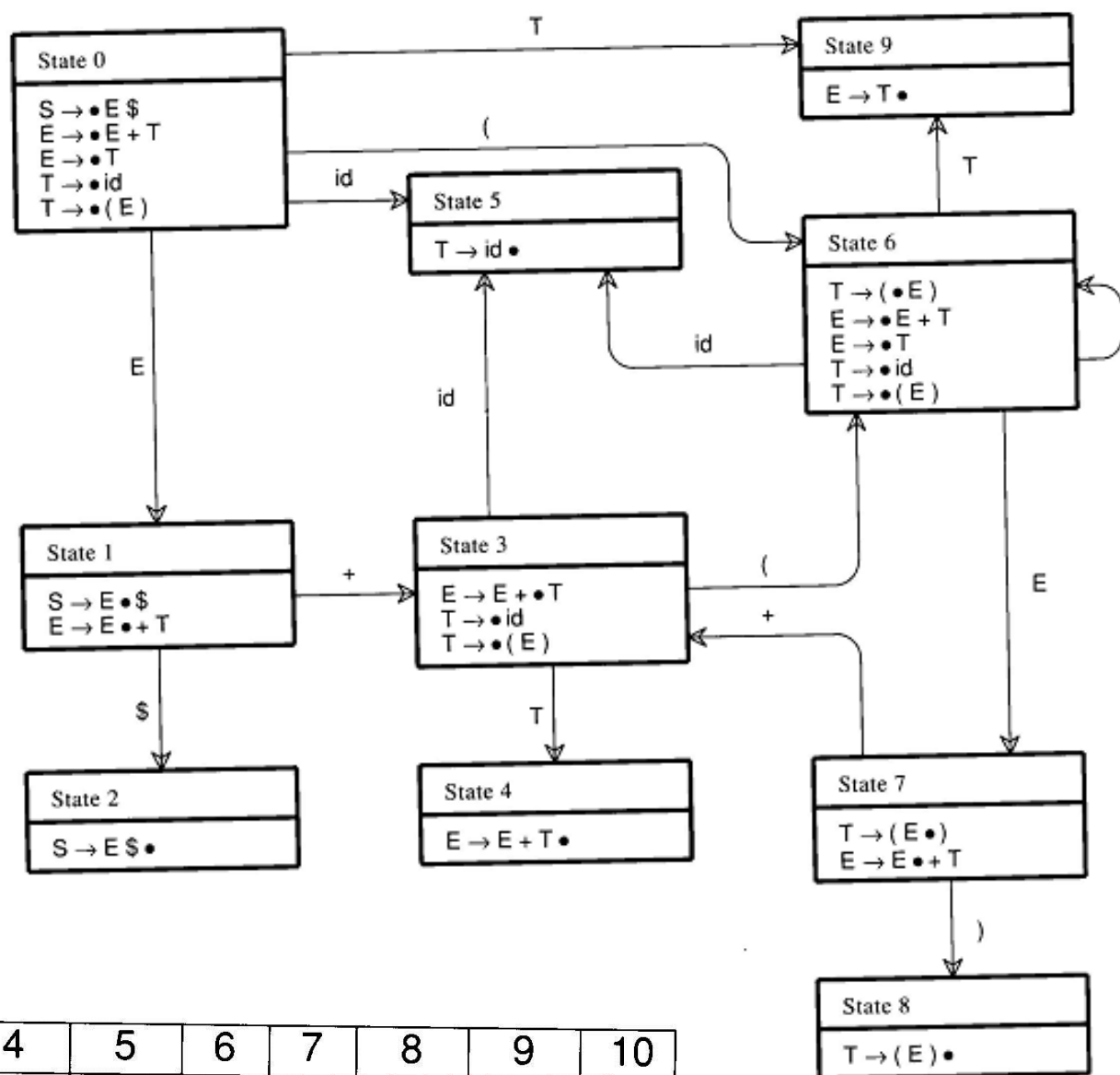
- Consider G_1

$S \rightarrow E\$$

$E \rightarrow E+T \mid T$

$T \rightarrow ID|(E)$

CFSM for $G_1 \Rightarrow$



State:	0	1	2	3	4	5	6	7	8	9	10
Action:	S	S	A	S	R2	R4	S	S	R5	R3	

Figure 6.12 action Table for G_1

LR(0) Parsing

- Any state $s \in S_0$ for which $|P(s)| > 1$ is said to be *inadequate*
- Two kinds of parser conflicts create inadequacies in configuration sets
 - Shift-reduce conflicts
 - Reduce-reduce conflicts

LR(0) Parsing

- It is easy to introduce inadequacies in CFSM states
 - Hence, few real grammars are LR(0). For example,
 - Consider λ -productions
 - The only possible configuration involving a λ -production is of the form $A \rightarrow \lambda \bullet$
 - However, if A can generate any terminal string other than λ , then a shift action must also be possible ($\text{First}(A)$)
 - LR(0) parser will have problems in handling operator precedence properly

LR(1) Parsing

- An LR(1) configuration, or item is of the form
 - $A \rightarrow X_1 X_2 \dots X_i \bullet X_{i+1} \dots X_j, l$ where $l \in V_t \cup \{\lambda\}$
 - The look ahead component l represents a possible lookahead after the entire right-hand side has been matched
 - The λ appears as lookahead only for the augmenting production because there is no lookahead after the endmarker

LR(1) Parsing

- We use the following notation to represent the **set** of LR(1) configurations that shared the same dotted production

$$\begin{aligned} &A \rightarrow X_1 X_2 \dots X_i \bullet X_{i+1} \dots X_j, \{l_1 \dots l_m\} \\ &= \{A \rightarrow X_1 X_2 \dots X_i \bullet X_{i+1} \dots X_j, l_1\} \cup \\ &\quad \{A \rightarrow X_1 X_2 \dots X_i \bullet X_{i+1} \dots X_j, l_2\} \cup \\ &\quad \dots \\ &\quad \{A \rightarrow X_1 X_2 \dots X_i \bullet X_{i+1} \dots X_j, l_m\} \end{aligned}$$

LR(1) Parsing

- There are many more distinct LR(1) configurations than LR(0) configurations.
- In fact, the major difficulty with LR(1) parsers is not their power but rather finding ways to represent them in storage-efficient ways.

LR(1) Parsing

- Parsing begins with the configuration
 - **closure1**(**{S**→•**α \$**, **{λ}**)

```
configuration_set closure1(configuration_set s)
{
    configuration_set s' = s;

    do {
        if (B → δ • Aρ, l ∈ s' for A ∈ Vn) {
            /*
             * Predict productions with A as the
             * left-hand side. Possible lookaheads
             * are First(ρl)
             */
            Add all configurations of the form A → • γ, u,
              where u ∈ First(ρl), to s'
        }
    } while (more new configurations can be added)
    return s';
}
```

Figure 6.13 An Algorithm to Close LR(1) Configuration Sets

LR(1) Parsing

- Consider G_1

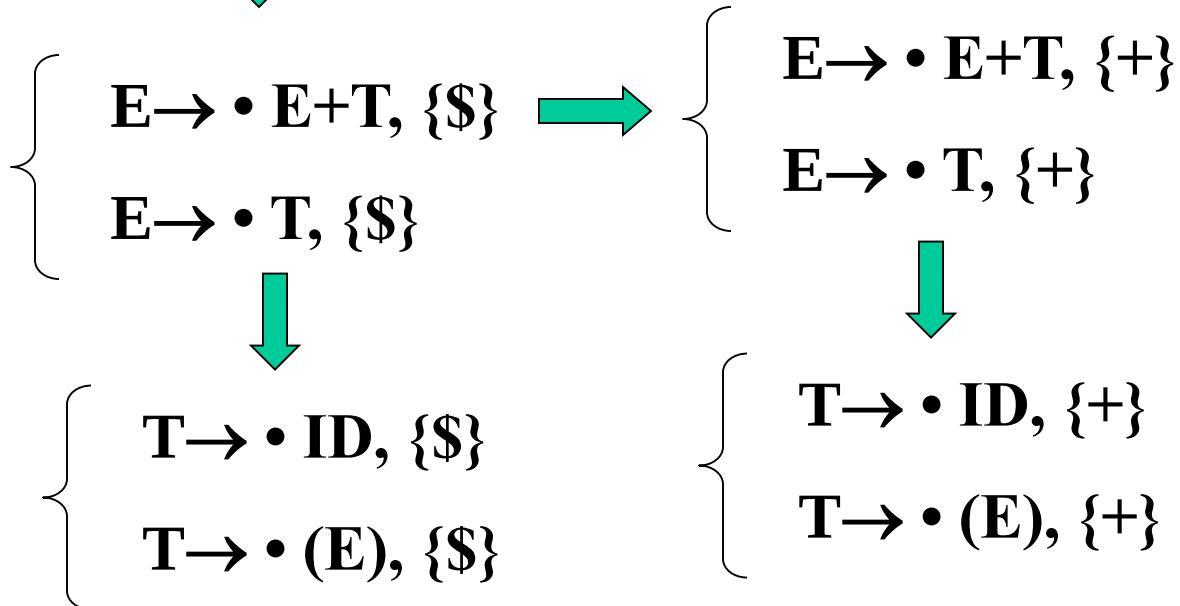
$S \rightarrow E\$$

$E \rightarrow E+T \mid T$

$T \rightarrow ID \mid (E)$

- $\text{closure}_1(S \rightarrow \bullet E\$, \{\lambda\})$

$S \rightarrow \bullet E\$, \{\lambda\}$



$\text{closure}_1(S \rightarrow \bullet E\$, \{\lambda\}) =$

{

$S \rightarrow \bullet E\$, \{\lambda\};$

$E \rightarrow \bullet E+T, \{\$+\}$

$E \rightarrow \bullet T, \{\$+\}$

$T \rightarrow \bullet ID, \{\$+\}$

$T \rightarrow \bullet (E), \{\$+\}$

}

LR(1) Parsing

- Given an LR(1) configuration set s , we compute its successor, s' , under a symbol X
 - $\text{go_to1}(s, X)$

```
configuration_set go_to1(configuration_set s, symbol X)
{
     $s_b = \emptyset$ ;
    for (each configuration  $c \in s$ )
        if ( $c$  is of the form  $A \rightarrow \beta \bullet X \gamma, l$ )
            Add  $A \rightarrow \beta X \bullet \gamma, l$  to  $s_b$ ;

    /*
     * That is, we advance the  $\bullet$  past the symbol  $X$ ,
     * if possible. Configurations not having a
     * dot preceding an  $X$  are not included in  $s_b$ .
     */

    /* Add new predictions to  $s_b$  via closure1. */
    return closure1( $s_b$ );
}
```

Figure 6.14 An Algorithm to Compute the LR(1) `go_to` Function

LR(1) Parsing

- We can build a finite automation that is analogue of the LR(0) CFSM
 - LR(1) FSM, LR(1) machine
- The relationship between CFSM and LR(1) machine
 - By merging LR(1) machine's configuration sets, we can obtain CFSM

• G_3

$S \rightarrow E\$$

$E \rightarrow E+T|T$

$T \rightarrow T*P|P$

$P \rightarrow ID|(E)$

```
void build_LR1(void)
{
    Create the Start State of the FSM; Label it with  $s_0$ 
    Put  $s_0$  into an initially empty set, S.
    while (S is nonempty) {
        Remove a configuration set s from S;
        /* Consider both terminals and nonterminals */
        for (X in Symbols) {
            if (go_tol(s,X)  $\neq \emptyset$ ) {
                if (go_tol(s,X) does not label a FSM state) {
                    Create a new FSM state and label it
                    with go_tol(s,X);
                    Put go_tol(s,X) into S;
                }
                Create a transition under X from the
                state s labels to the state
                go_tol(s,X) labels;
            }
        }
    }
}
```

Figure 6.15 An Algorithm to Build an LR(1) FSM

- G_3
- $S \rightarrow E\$$
- $E \rightarrow E+T \mid T$
- $T \rightarrow T*P \mid P$
- $P \rightarrow ID \mid (E)$

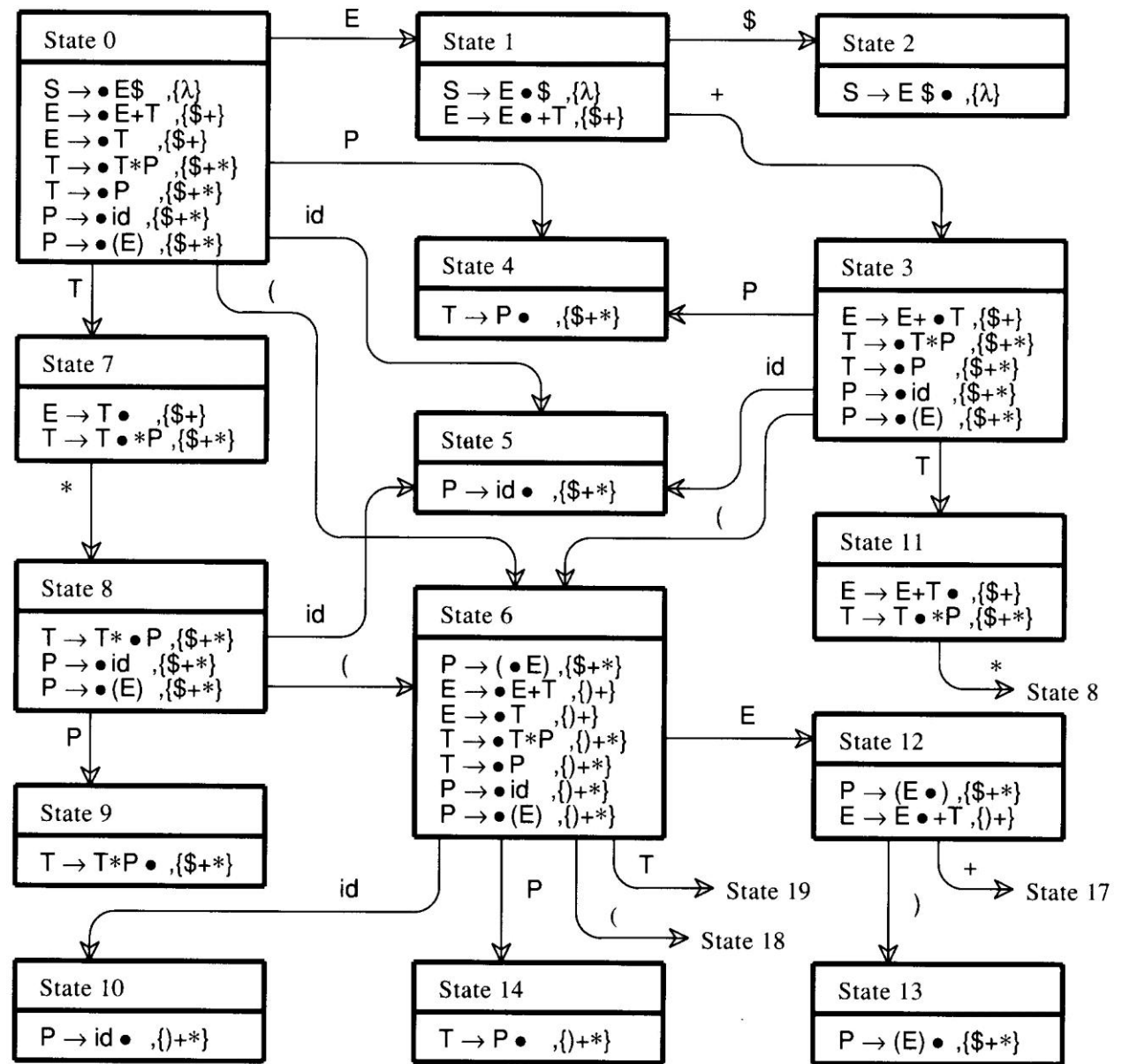


Figure 6.16 LR(1) Machine for G_3

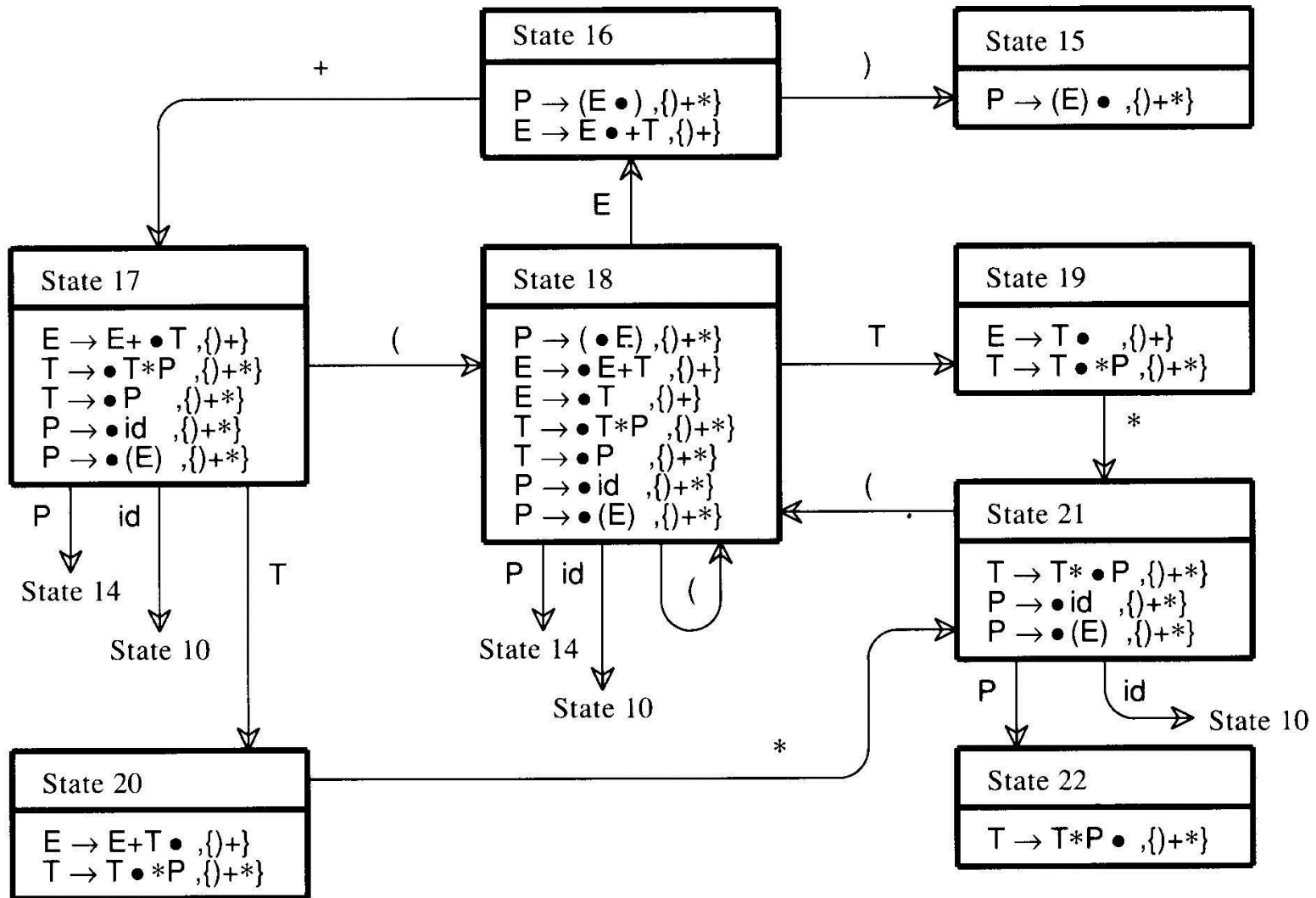


Figure 6.16 (continued)

LR(1) Parsing

- The go_to table used to drive an LR(1) is extracted directly from the LR(1) machine

```
int ** build_go_to_table(finite_automaton CFSM)
{
    const int N = num_states(CFSM);
    int **tab;

    Dynamically allocate a table of dimension
    N × num_symbols(CFSM) to represent
    the go_to table and assign it to tab;

    Number the states of CFSM from 0 to N-1,
    with the Start State labeled 0;

    for (S = 0; S <= N - 1; S++) {
        /* Consider both terminals and nonterminals. */
        for (X in Symbols) {
            if (State S has a transition under X
                to some state T)
                tab[S][X] = T;
            else
                tab[S][X] = EMPTY;
        }
    }
    return tab;
}
```

Figure 6.9 An Algorithm to Build the LR(0) go_to Table

LR(1) Parsing

- Action table is extracted directly from the configuration sets of the LR(1) machine
- A projection function, P
 - $P : S_1 \times V_t \rightarrow 2^Q$
 - S_1 be the set of LR(1) machine states
- $P(s,a) = \{ \text{Reduce}_i \mid B \rightarrow \rho \bullet, a \in s \text{ and production } i \text{ is } B \rightarrow \rho \} \cup (\text{if } A \rightarrow \alpha \bullet a\beta, b \in s \text{ Then } \{ \text{Shift} \} \text{ Else } \emptyset)$

LR(1) Parsing

- G is LR(1) if and only if
 - $\forall s \in S_1 \ \forall a \in V_t \ |P(s,a)| \leq 1$
- If G is LR(1), the **action** table is trivially extracted from P
 - $P(s, \$) = \{\text{Shift}\} \Rightarrow \text{action}[s][\$] = \text{Accept}$
 - $P(s, a) = \{\text{Shift}\}, a \neq \$ \Rightarrow \text{action}[s][a] = \text{Shift}$
 - $P(s, a) = \{\text{Reduce}_i\}, \Rightarrow \text{action}[s][a] = \text{Reduce}_i$
 - $P(s, a) = \emptyset \Rightarrow \text{action}[s][a] = \text{Error}$

State	Lookahead					
	+	*	ID	(	)	\$
0			S	S		
1	S					A
2						
3			S	S		
4	R5	R5				R5
5	R6	R6				R6
6			S	S		
7	R3	S				R3
8			S	S		
9	R4	R4				R4
10	R6	R6			R6	
11	R2	S				R2
12	S				S	
13	R7	R7				R7
14	R5	R5			R5	
15	R7	R7			R7	
16	S				S	
17			S	S		
18			S	S		
19	R3	S			R3	
20	R2	S			R2	
21			S	S		
22	R4	R4			R4	

Figure 6.17 LR(1) **action** Function for G_3

SLR(1) Parsing

- LR(1) parsers are the most powerful class of shift-reduce parsers, using a single lookahead
 - LR(1) grammars exist for virtually all programming languages
 - LR(1)'s problem is that the LR(1) machine contains so many states that the `go_to` and action tables become prohibitively large

SLR(1) Parsing

- In reaction to the space inefficiency of LR(1) tables, computer scientists have devised parsing techniques that are almost as powerful as LR(1) but that require far smaller tables
 1. One is to start with the CFSM, and then add lookahead after the CFSM is build
 - SLR(1)
 2. The other approach to reducing LR(1)'s space inefficiencies is to merger inessential LR(1) states
 - LALR(1)

SLR(1) Parsing

- SLR(1) stands for *Simple LR(1)*
 - One-symbol lookahead
 - Lookaheads are not built directly into configurations but rather are added after the LR(0) configuration sets are built
 - An SLR(1) parser will perform a reduce action for configuration $\mathbf{B} \rightarrow \rho \bullet$ if the lookahead symbol is in the set $\mathbf{Follow}(\mathbf{B})$

SLR(1) Parsing

- The SLR(1) projection function, from CFSM states,
 - $P : S_0 \times V_t \rightarrow 2^Q$
 - $P(s, a) = \{ \text{Reduce}_i \mid B \rightarrow \rho \bullet, a \in \text{Follow}(B) \text{ and production } i \text{ is } B \rightarrow \rho \} \cup \{ \text{Shift} \mid \text{if } A \rightarrow \alpha \bullet a \beta \in s \text{ for } a \in V_t \}$

SLR(1) Parsing

- G is SLR(1) if and only if
 - $\forall s \in S_0 \ \forall a \in V_t \ |P(s,a)| \leq 1$
- If G is SLR(1), the **action** table is trivially extracted from P
 - $P(s, \$) = \{\text{Shift}\} \Rightarrow \text{action}[s][\$] = \text{Accept}$
 - $P(s, a) = \{\text{Shift}\}, a \neq \$ \Rightarrow \text{action}[s][a] = \text{Shift}$
 - $P(s, a) = \{\text{Reduce}_i\}, \Rightarrow \text{action}[s][a] = \text{Reduce}_i$
 - $P(s, a) = \emptyset \Rightarrow \text{action}[s][a] = \text{Error}$
- Clearly SLR(1) is a proper superset of LR(0)

SLR(1) Parsing

- Consider G_3
 - It is LR(1) but not LR(0)
 - See states 7,11
 - $\text{Follow}(E) = \{\$, +,)\}$

$S \rightarrow E\$$

$E \rightarrow E+T \mid T$

$T \rightarrow T*P \mid P$

$P \rightarrow ID \mid (E)$

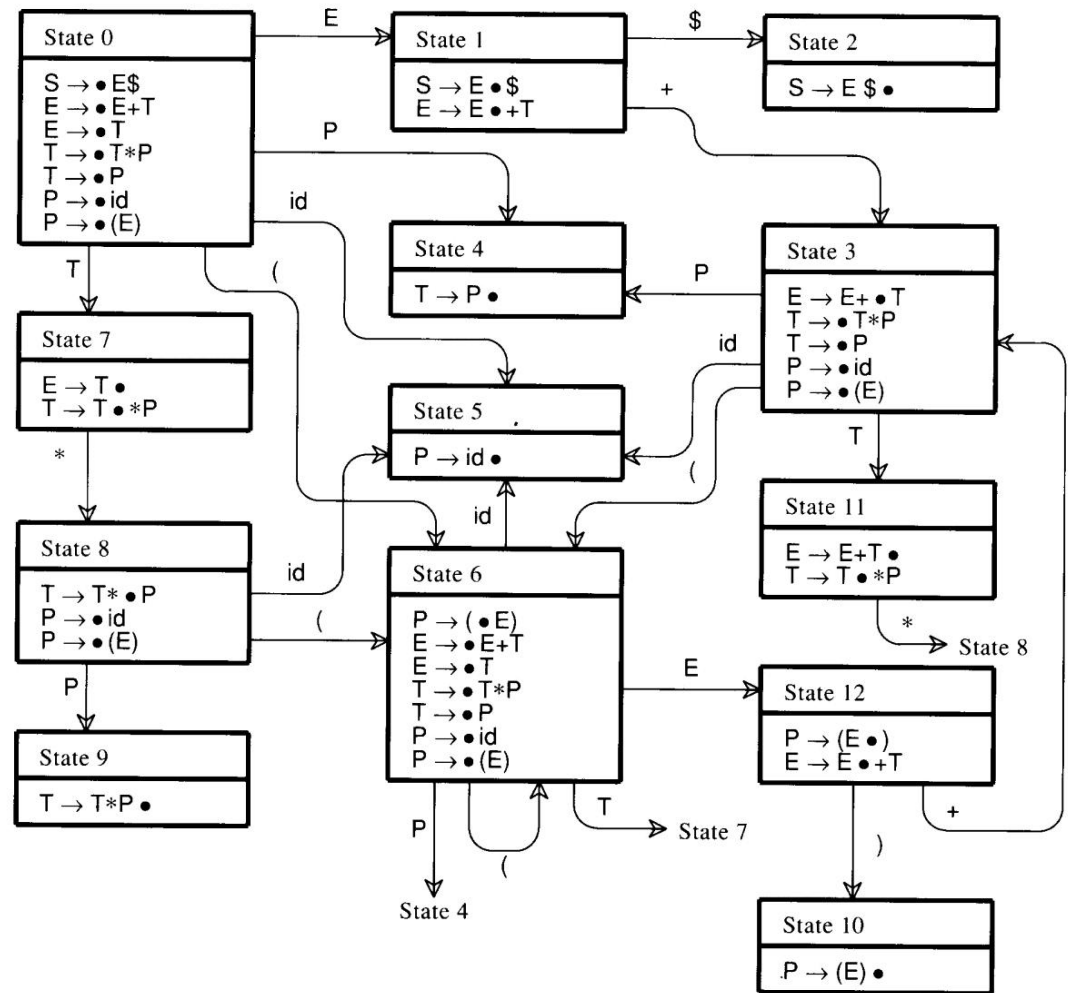


Figure 6.18 CFSM for G_3

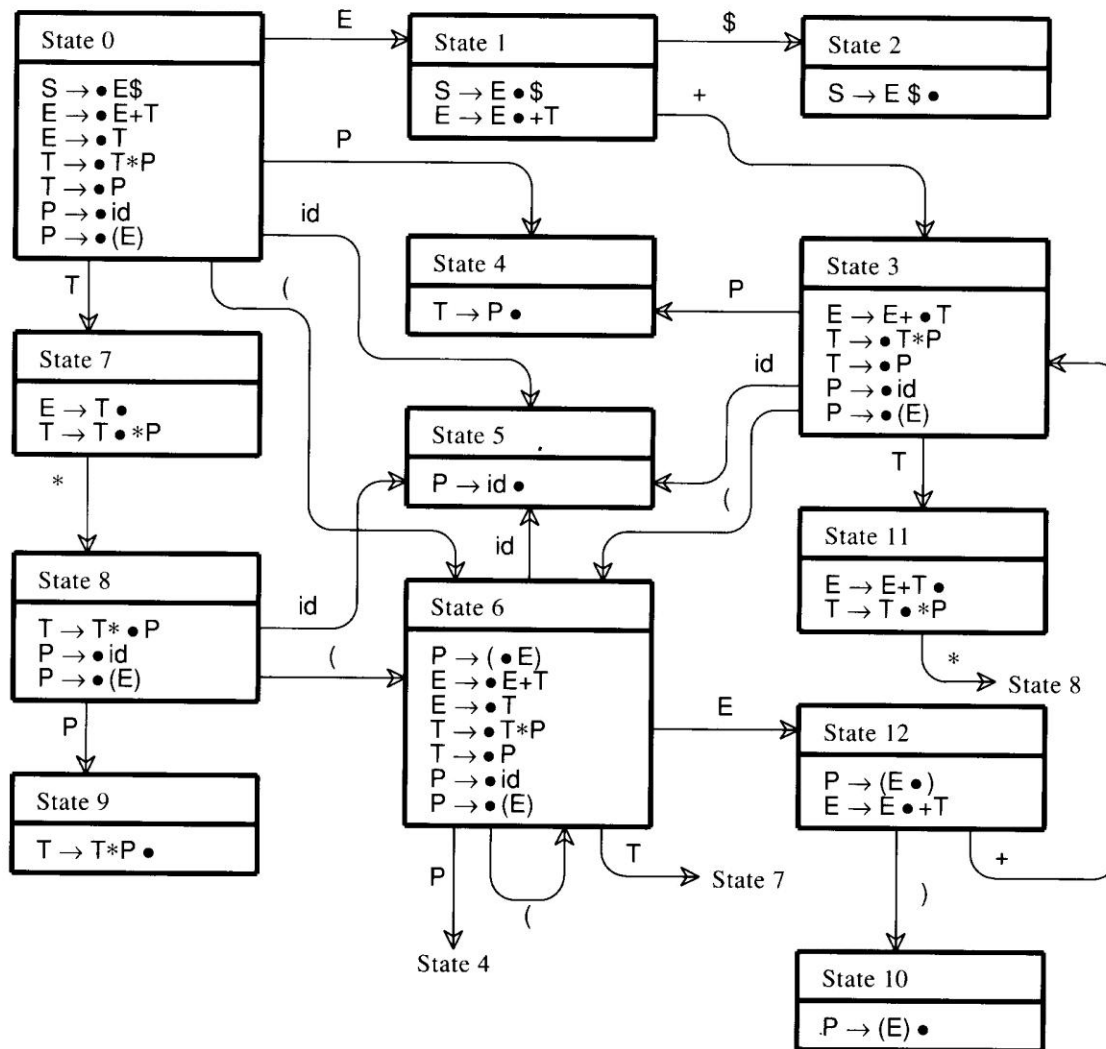


Figure 6.18 CFSM for G_3

$S \rightarrow E \$$
 $E \rightarrow E + T \mid T$
 $T \rightarrow T * P \mid P$
 $P \rightarrow ID \mid (E)$

State	Lookahead					
	+	*	ID	(	)	\$
0			S	S		
1	S					A
2						
3			S	S		
4	R5	R5			R5	R5
5	R6	R6			R6	R6
6			S	S		
7	R3	S			R3	R3
8			S	S		
9	R4	R4			R4	R4
10	R7	R7			R7	R7
11	R2	S			R2	R2
12	S				S	

Figure 6.19 SLR(1) **action** Function for G_3

Limitations of the SLR(1) Technique

- The use of Follow sets to estimate the lookaheads that predict reduce actions is less precise than using the exact lookaheads incorporated into LR(1) configurations
 - Consider G_4

Elem $\rightarrow$ (List, Elem)

Elem $\rightarrow$ Scalar

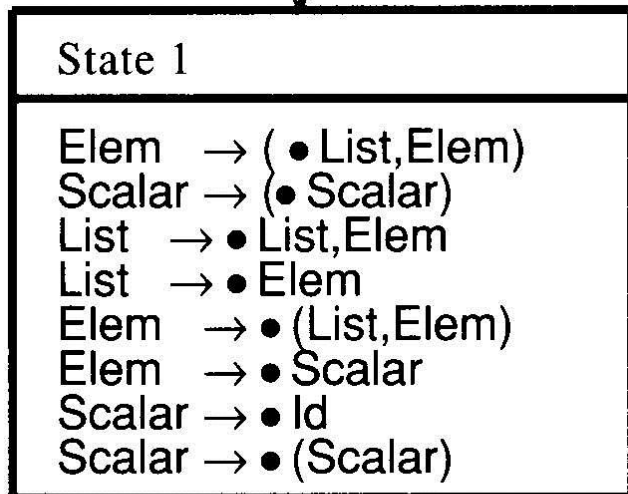
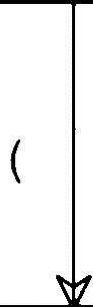
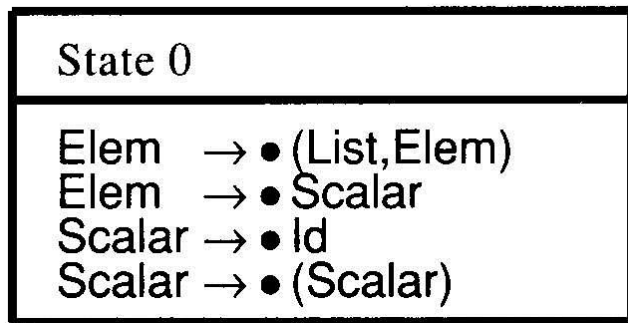
List $\rightarrow$ List, Elem

List $\rightarrow$ Elem

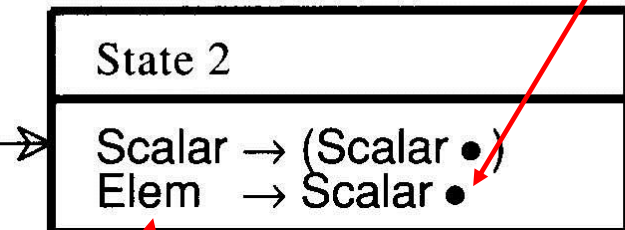
Scalar $\rightarrow$ ID

Scalar $\rightarrow$ (Scalar)

Follow(Elem) = {“(”, “)”, “,”, “;”, “.”, ...}



Scalar



LR(1) lookahead for
Elem $\rightarrow$ **Scalar** $\bullet$ is “,”

$Fellow(Elem) = \{ “)”, “,”, “;”, \dots \}$

Figure 6.20 Part of the CFSM for G_4

LALR(1)

- LALR(1) parsers can be built by first constructing an LR(1) parser and then merging states
 - An LALR(1) parser is an LR(1) parser in which all states that differ only in the lookahead components of the configurations are *merged*
 - LALR is an acronym for *L*ook *A*head LR

The core of a configuration

State 3
$E \rightarrow E + \bullet T, \{\$+\}$
$T \rightarrow \bullet T * P, \{\$+\}$
$T \rightarrow \bullet P, \{\$+\}$
$P \rightarrow \bullet id, \{\$+\}$
$P \rightarrow \bullet (E), \{\$+\}$

State 17
$E \rightarrow E + \bullet T, \{()\}$
$T \rightarrow \bullet T * P, \{()\}$
$T \rightarrow \bullet P, \{()\}$
$P \rightarrow \bullet id, \{()\}$
$P \rightarrow \bullet (E), \{()\}$

- The core of the above two configurations is the same

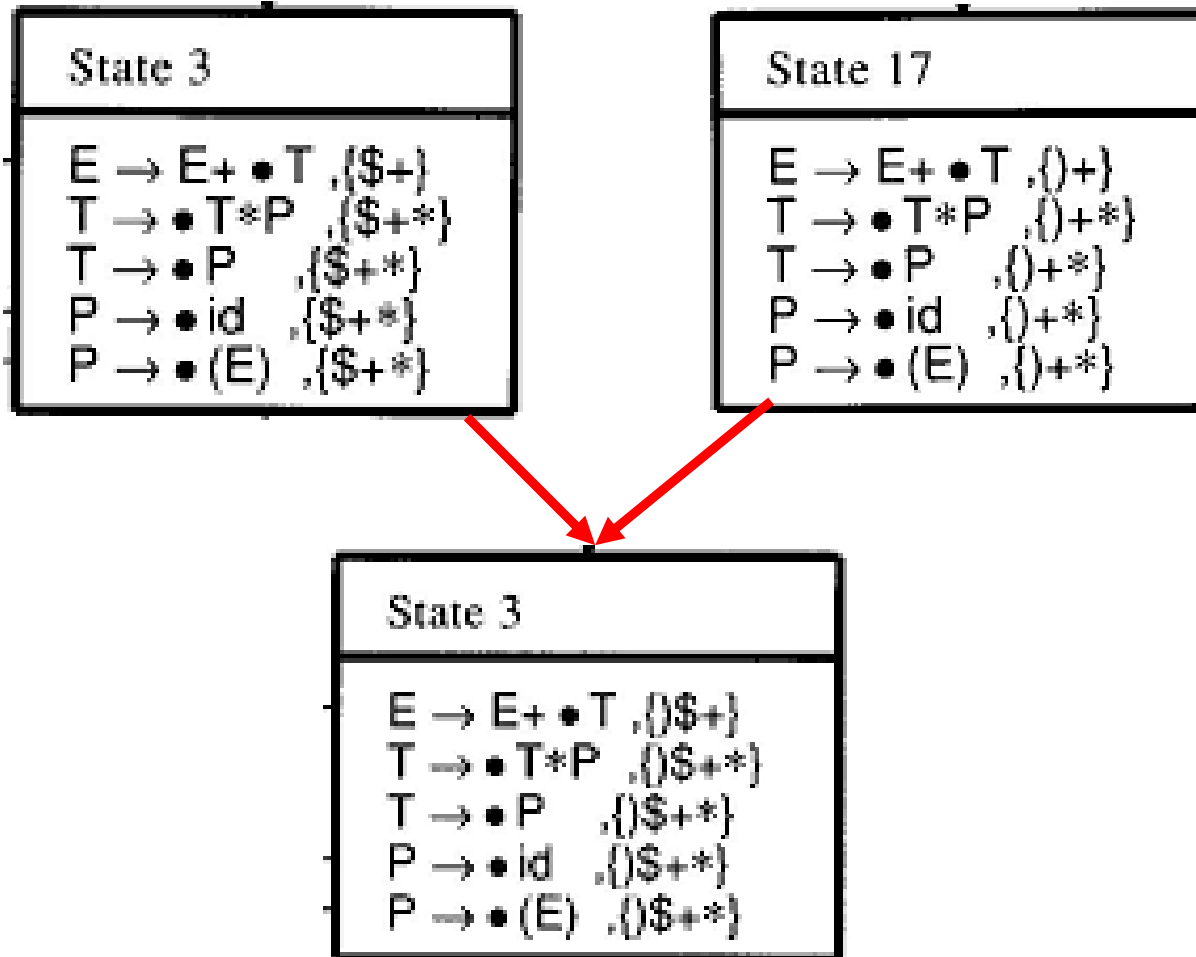
$E \rightarrow E + \bullet T$
 $T \rightarrow \bullet T * P$
 $T \rightarrow \bullet P$
 $P \rightarrow \bullet id$
 $P \rightarrow \bullet (E)$

States Merge

$E \rightarrow E + \bullet T$
 $T \rightarrow \bullet T * P$
 $T \rightarrow \bullet P$
 $P \rightarrow \bullet id$
 $P \rightarrow \bullet (E)$

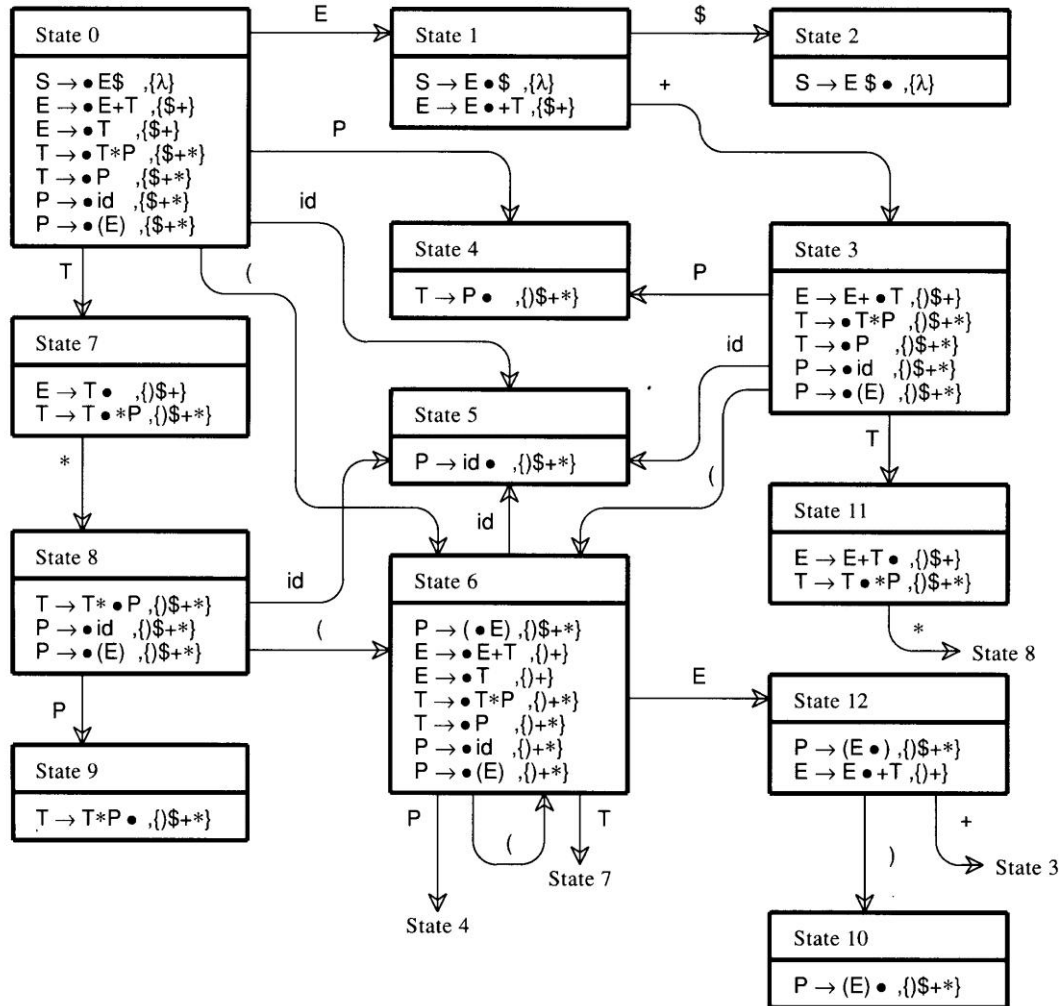
s'

- $\text{Cognate}(s') = \{c | c \in s, \text{core}(s) = s'\}$



LALR(1)

- LALR(1) machine



LALR(1) Cognate State	LR(1) States with Common Core
State 0	State 0
State 1	State 1
State 2	State 2
State 3	State 3, State 17
State 4	State 4, State 14
State 5	State 5, State 10
State 6	State 6, State 18
State 7	State 7, State 19
State 8	State 8, State 21
State 9	State 9, State 22
State 10	State 13, State 15
State 11	State 11, State 20
State 12	State 12, State 16

Figure 6.21 Cognate States for G_3

Figure 6.22 LALR(1) Machine for G_3

LALR(1)

- The CFSM state is transformed into its LALR(1) Cognate
 - $P : S_0 \times V_t \rightarrow 2^Q$
 - $P(s, a) = \{ \text{Reduce}_i \mid B \rightarrow \rho \bullet, a \in \text{Cognate}(s) \text{ and production } i \text{ is } B \rightarrow \rho \} \cup (\text{if } A \rightarrow \alpha \bullet a \beta \in s \text{ Then } \{ \text{Shift} \} \text{ Else } \emptyset)$

LALR(1) Parsing

- G is LALR(1) if and only if
 - $\forall s \in S_0 \ \forall a \in V_t \ |P(s,a)| \leq 1$
- If G is LALR(1), the **action** table is trivially extracted from P
 - $P(s, \$) = \{\text{Shift}\} \Rightarrow \text{action}[s][\$] = \text{Accept}$
 - $P(s, a) = \{\text{Shift}\}, a \neq \$ \Rightarrow \text{action}[s][a] = \text{Shift}$
 - $P(s, a) = \{\text{Reduce}_i\}, \Rightarrow \text{action}[s][a] = \text{Reduce}_i$
 - $P(s, a) = \emptyset \Rightarrow \text{action}[s][a] = \text{Error}$

LALR(1) Parsing

- Consider G_5

$\langle \text{stmt} \rangle \rightarrow \text{ID}$

$\langle \text{stmt} \rangle \rightarrow \langle \text{var} \rangle := \langle \text{expr} \rangle$

$\langle \text{var} \rangle \rightarrow \text{ID}$

$\langle \text{var} \rangle \rightarrow \text{ID} [\langle \text{expr} \rangle]$

$\langle \text{expr} \rangle \rightarrow \langle \text{var} \rangle$

- Assume statements are separated by ;'s, the grammar is not SLR(1) because
; $\in \text{Follow}(\langle \text{stmt} \rangle)$ and
; $\in \text{Follow}(\langle \text{var} \rangle)$, since $\langle \text{expr} \rangle \rightarrow \langle \text{var} \rangle$

LALR(1) Parsing

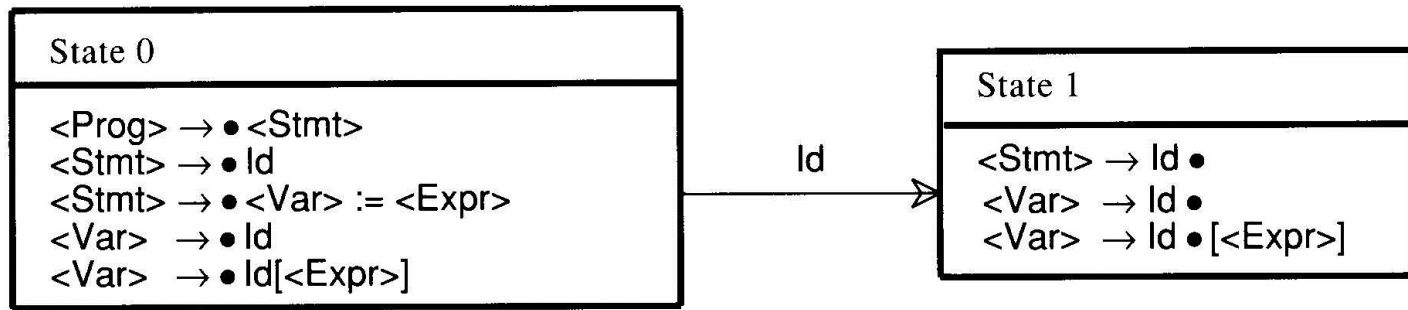


Figure 6.23 Part of the CFSM for G_5

- However, in LALR(1), if we use $\langle \text{var} \rangle \rightarrow \text{ID}$ the next symbol must be $:=$
so $\text{action}[1, :=] = \text{reduce}(\langle \text{var} \rangle \rightarrow \text{ID})$
 $\text{action}[1, ;] = \text{reduce}(\langle \text{stmt} \rangle \rightarrow \text{ID})$
 $\text{action}[1, [] = \text{shift}$
- There is no conflict.

LALR(1) Parsing

- A common technique to put an LALR(1) grammar into SLR(1) form is to introduce a new nonterminal whose global (I.e. SLR) lookaheads more nearly correspond to LALR's exact lookaheads
 - $\text{Follow}(\langle \text{lhs} \rangle) = \{ := \}$

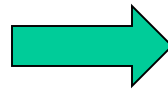
$\langle \text{stmt} \rangle \rightarrow \text{ID}$

$\langle \text{stmt} \rangle \rightarrow \langle \text{var} \rangle := \langle \text{expr} \rangle$

$\langle \text{var} \rangle \rightarrow \text{ID}$

$\langle \text{var} \rangle \rightarrow \text{ID} [\langle \text{expr} \rangle]$

$\langle \text{expr} \rangle \rightarrow \langle \text{var} \rangle$



$\langle \text{stmt} \rangle \rightarrow \text{ID}$

$\langle \text{stmt} \rangle \rightarrow \langle \text{lhs} \rangle := \langle \text{expr} \rangle$

$\langle \text{lhs} \rangle \rightarrow \text{ID}$

$\langle \text{lhs} \rangle \rightarrow \text{ID} [\langle \text{expr} \rangle]$

$\langle \text{var} \rangle \rightarrow \text{ID}$

$\langle \text{var} \rangle \rightarrow \text{ID} [\langle \text{expr} \rangle]$

$\langle \text{expr} \rangle \rightarrow \langle \text{var} \rangle$

LALR(1) Parsing

- At times, it is the CFSM itself that is at fault.

$S \rightarrow (\text{Exp1})$

$S \rightarrow [\text{Exp1}]$

$S \rightarrow (\text{Exp1})$

$S \rightarrow [\text{Exp1}]$

$\langle \text{Exp1} \rangle \rightarrow \text{ID}$

$\langle \text{Exp2} \rangle \rightarrow \text{ID}$

- A different expression nonterminal is used to allow error or warning diagnostics

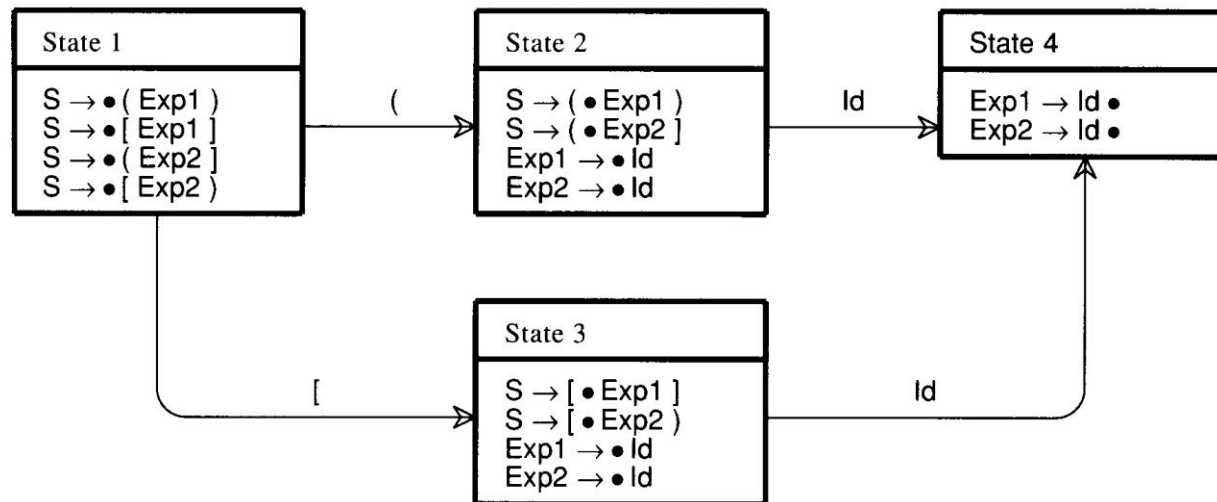


Figure 6.24 Part of the CFSM for Grammar G_6

Building LALR(1) Parsers

- In the definition of LALR(1)
 - An LR(1) machine is first built, and then its states are merged to form an automaton identical in structure to the CFSM
 - May be quite inefficient
 - An alternative is to build the CFSM first.
 - Then LALR(1) lookaheads are “*propagated*” from configuration to configuration

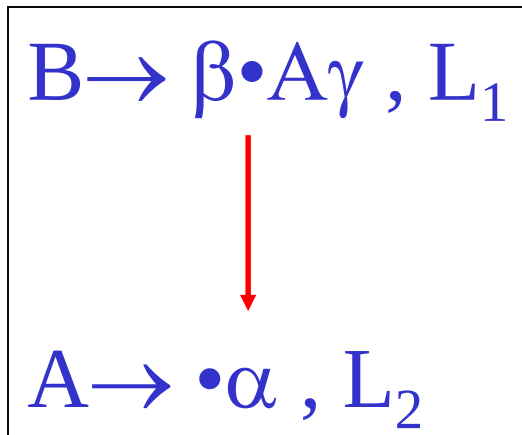
Building LALR(1) Parsers

- Propagate links:
 - **Case 1**: one configuration is created from another in a previous state via a shift operation



Building LALR(1) Parsers

- Propagate links:
 - **Case 2**: one configuration is created as the result of a closure or prediction operation on another configuration



$$L_2 = \{ x \mid x \in \text{First}(\gamma t) \text{ and } t \in L_1 \}$$

Building LALR(1) Parsers

- **Step 1:** After the CFSM is built, we can create all the necessary propagate links to transmit lookaheads from one configuration to another
- **Step 2:** spontaneous lookaheads are determined
 - By including in L_2 , for configuration $A \rightarrow \bullet \alpha, L_2$, all spontaneous lookaheads induced by configurations of the form $B \rightarrow \beta \bullet A \gamma, L_1$
 - These are simply the non- λ values of $\text{First}(\gamma)$
- **Step 3:** Then, propagate lookaheads via the propagate links
 - See figure 6.25

Building LALR(1) Parsers

```
while (stack is not empty)
{
    pop top item, assign its components to (s,c,L)

    if (configuration c in state s
        has any propagate links) {
        Try, in turn, to add L to the lookahead set of
        each configuration so linked.
        for (each configuration  $\bar{c}$  in state  $\bar{s}$ 
            to which L is added)
            Push ( $\bar{s}, \bar{c}, L$ ) onto the stack.
    }
}
```

Figure 6.25 LALR(1) Lookahead Propagation Algorithm

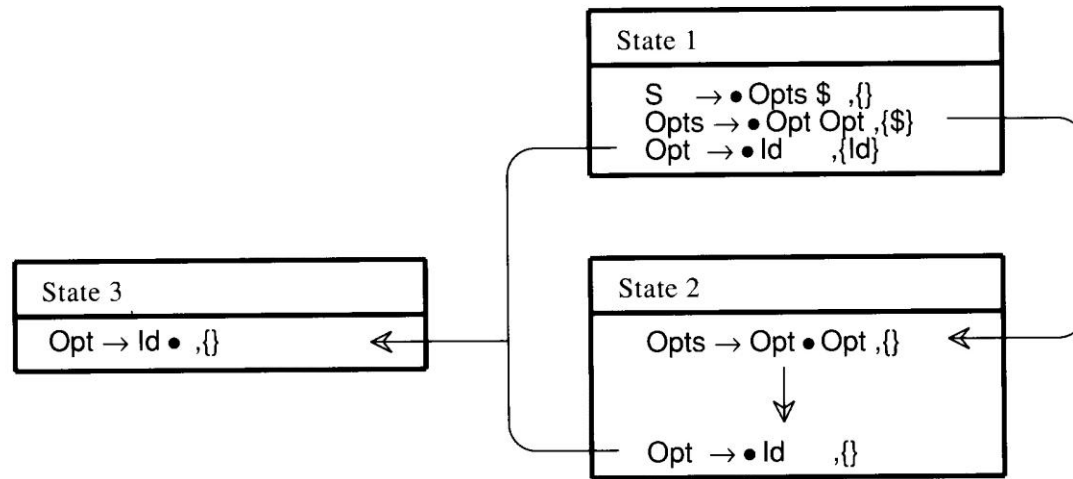


Figure 6.26 Part of CFSM for G_7 with Propagate Links

Step	Stack	Action
(1)	(s1,c2,\$), (s1,c3,ID)	Pop (s1,c2,\$) Add \$ to c1 in s2 Push (s2,c1,\$)
(2)	(s2,c1,\$), (s1,c3,ID)	Pop (s2,c1,\$) Add \$ to c2 in s2 Push (s2,c2,\$)
(3)	(s2,c2,\$), (s1,c3,ID)	Pop (s2,c2,\$) Add \$ to c1 in s3 Push (s3,c1,\$)
(4)	(s3,c1,\$), (s1,c3,ID)	Pop (s3,c1,\$) Nothing is added (no links)
(5)	(s1,c3,ID)	Pop (s1,c3,ID) Add ID to c1 in s3 Push (s3,c1,ID)
(6)	(s3,c1,ID)	Pop (s3,c1,ID) Nothing is added (no links)
(7)	Empty	Terminate algorithm

Figure 6.27 Example of Lookahead Propagation

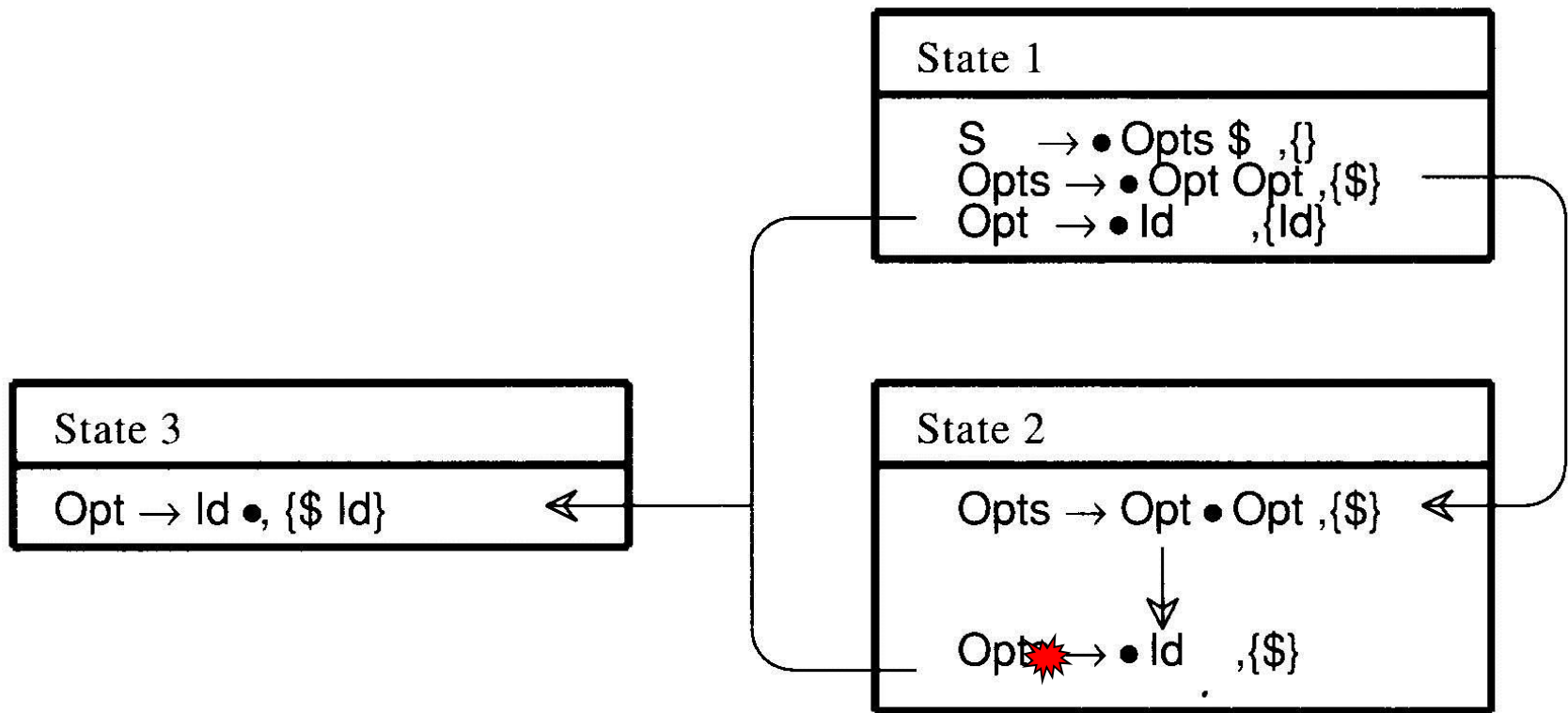


Figure 6.28 Part of CFSM for G_7 with Lookaheads Propagated

Building LALR(1) Parsers

- A number of LALR(1) parser generators use lookahead propagation to compute the parser action table
 - LALRGen uses the propagation algorithm
 - YACC examines each state repeatedly
- An intriguing alternative to propagating LALR lookaheads is to compute them as needed by doing a backward search through the CFSM
 - Read it yourself. P. 176, Para. 3

Calling Semantic Routines in Shift-Reduce Parsers

- Shift-reduce parsers can normally handle larger classes of grammars than LL(1) parsers, which is a major reason for their popularity
- Shift-reduce parsers are not predictive, so we cannot always be sure what production is being recognized until its entire right-hand side has been matched
 - The semantic routines can be invoked only after a production is recognized and reduced
 - Action symbols only at the extreme right end of a right-hand side

Calling Semantic Routines in Shift-Reduce Parsers

- Two common tricks are known that allow more flexible placement of semantic routine calls

- For example,

`<stmt> → if <expr> then <stmts> else <stmts> end if`

- We need to call semantic routines after the conditional expression *else* and *end if* are matched

- Solution: create new nonterminals that generate λ

`<stmt> → if <expr> <test cond>`

`then <stmts> <process then part>`

`else <stmts> end if`

`<test cond> →  $\lambda$`

`<process then part> →  $\lambda$`

Calling Semantic Routines in Shift-Reduce Parsers

- If the right-hand sides differ in the semantic routines that are to be called, the parser will be unable to correctly determine which routines to invoke

- Ambiguity will manifest. For example,

<stmt> → if <expr> <test cond1>

then <stmts> <process then part>

else <stmts> end if;

<stmt> → if <expr> <test cond2>

then <stmts> <process then part>

else <stmts> end if;

<test cond1> → λ

<test cond2> → λ

<process then part> → λ

Calling Semantic Routines in Shift-Reduce Parsers

- An alternative to the use of λ -generating nonterminals is to break a production into a number of pieces, with the breaks placed where semantic routines are required

$\langle \text{stmt} \rangle \rightarrow \langle \text{if head} \rangle \langle \text{then part} \rangle \langle \text{else part} \rangle$

$\langle \text{if head} \rangle \rightarrow \text{if } \langle \text{expr} \rangle$

$\langle \text{then part} \rangle \rightarrow \text{then } \langle \text{stmts} \rangle$

$\langle \text{else part} \rangle \rightarrow \text{then } \langle \text{stmts} \rangle \text{ end if;}$

- This approach can make productions harder to read but has the advantage that no λ -generating are needed