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# A generalized grid quorum strategy for *k*-mutual exclusion in distributed systems

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### Abstract

In the problem of k-mutual exclusion, concurrent access to shared resource or the critical section (CS) must be synchronized such that at any time at most k processes can access the CS. In this paper, we propose a generalized grid quorum strategy for k-mutual exclusion, which imposes a logical grid structure on the network. The quorum size is always equal to  $\lceil (M + 1)/(k + 1) \rceil \times \lceil (N + 1)/2 \rceil$ , where M is the number of rows and N is the number of columns in a grid. From our performance study, we show that the generalized grid quorum strategy can provide a good performance in terms of the quorum size and the availability. © 2001 Elsevier Science B.V. All rights reserved.

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## 1. Introduction

A distributed system consists of a collection of geographically dispersed autonomous nodes connected by a communication network. The nodes have no shared memory, no global clock, and communicated with one another by passing messages. Message propagation delay is finite but unpredictable.

The mutual exclusion problem was originally considered in centralized systems for the synchronization of exclusive access to the shared resource. In the problem of *k*-mutual exclusion, concurrent access to shared resource or the critical section (CS) must be synchronized such that at any time, at most *k* processes can access the CS, where  $k \ge 1$ . In distributed systems,

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the *k*-mutual exclusion problem arises in several interesting applications. For example, it could be used to monitor the number of processes in distributed systems that are allowed to perform a certain action, such as issuing broadcast messages. In such a case, the system may restrict the number of broadcasting processes so as to control the level of congestion. Another application is in the context of replicated databases that allow *bounded ignorance* [3]; i.e., transactions may specify that they do not need to be aware of the *k* most recent updates to the database.

To make distributed *k*-mutual exclusion protocols fault-tolerant to node and communication failures, many protocols based on the replica control strategies, for example, *coterie*, have been proposed. In [6], they extended the *majority quorum* strategy to *k*-majority *quorum* strategy; any permission from  $\lceil (total + 1)/(k + 1) \rceil$  (= W) nodes would form a quorum for

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k-mutual exclusion, when total is the number of nodes in the system, and the following conditions must be satisfied:  $k \times W \leq total$  and  $(k + 1) \times W > total$  [8]. In [5], they proposed a *cohort quorum* for k-mutual exclusion based on a *cohort structure*, Coh(k, l), which has l pairwise disjoint cohorts with the first cohort having k members and the others having more than (2k - 2) members. This strategy may arrange nodes according to their up-probabilities, which suits the heterogeneous system that has nodes of different up-probabilities. In [1], they partition total nodes into k classes with each class using any traditional approach to enforce 1-mutual exclusion. When the traditional approach is the majority quorum strategy, the constructed quorums will be called DIV of majority quorums.

To reduce the overhead of achieving *k*-mutual exclusion while supporting fault tolerance, in this paper, we propose a strategy called *generalized grid quorum* for *k*-mutual exclusion, which imposes a logical grid structure [2] on the network. The quorum size constructed from this strategy is always equal to  $\lceil (M + 1)/(k + 1) \rceil \times \lceil (N + 1)/2 \rceil$  where *M* is the number of rows and *N* is the number of columns in a grid (i.e.,  $total = M \times N$ ). From our performance study, we show that the generalized grid quorum strategy can provide a good performance in terms of the quorum size and the availability.

The rest of the paper is organized as follows. Section 2 describes the background in this paper. In Section 3, we present the generalized grid quorums. In Section 4, we make a comparison of the generalized grid quorum strategy with the k-majority, cohorts, and DIV strategies. Finally, Section 5 gives a conclusion.

## 2. Background

A distributed system is a collection of nodes that may communicate with each other by exchanging messages. k-mutual exclusion strategies concern themselves with controlling the nodes such that at most k nodes can simultaneously access their critical sections. Such strategies can be used to coordinate the sharing of a resource that can be allocated to no more than k nodes at a time [1,5,6,8]. **Definition 1.** A *k*-coterie *C* is a family of nonempty subsets of an underlying set *U*, which is a set containing all system nodes 1, 2, ..., total. Each member *Q* in *C* is called a quorum, and the following properties should hold for the quorums [6]:

- (1) The non-intersection property. For any h(< k) pairwise disjoint quorums  $Q_1, \ldots, Q_h$  in *C*, there exists one quorum  $Q_{h+1}$  in *C* such that  $Q_1, \ldots, Q_{h+1}$  are pairwise disjoint.
- (2) The intersection property. There are no m, m > k, pairwise disjoint quorums in C (i.e., there are at most k pairwise disjoint quorums in C).
- (3) The minimality property. There are no two quorums Q<sub>i</sub> and Q<sub>j</sub> in C such that Q<sub>i</sub> is a super set of Q<sub>i</sub> where i ≠ j.

By the non-intersection property, if there exists one unoccupied entry of the critical section, then some node that waits for entering the critical section can proceed. The intersection property assures that no more than *k* nodes can form quorums simultaneously, so no more than *k* nodes can access the critical section at the same time. Again, the minimality property for the *k*-coterie is for the enhancement of efficiency. For example,  $\{\{1, 2\}, \{3, 4\}, \{1, 3\}, \{2, 4\}\}$  is a 2-coterie under  $U = \{1, 2, 3, 4\}$ .

# 3. Generalized grid quorums

In this section, we present a *generalized grid quo*rum for k-mutual exclusion based on a grid  $M \times N$ structure imposed in the system. Between rows, we apply the k-majority strategy, and inside each row, we apply the majority strategy.

#### 3.1. Definition

In this section, we give the definition of the generalized grid quorums.

**Definition 2** (*A generalized grid quorum*). The generalized grid quorum strategy logically organizes the nodes in a system as an  $M \times N$  (= *total*) grid structure, where *M* and *N* denote the number of nodes in a row and column, respectively,  $M \ge k$  and  $N \ge 1$ . We number the rows and columns of the grid structure as 0, 1, 2, ..., (M - 1) and 1, 2, ..., N, respectively. For



Fig. 1. A  $4 \times 3$  grid for 2-mutual exclusion with *total* = 12.

the nodes in each row  $row_i$ ,  $0 \le i \le (M-1)$ , let  $R_i$  be any one of the majority quorums for 1-mutual exclusion. A generalized grid quorum Q (for *k*-mutual exclusion) contains any  $\lceil (M+1)/(k+1) \rceil$  (= *W*) quorums from  $R_0, R_1, \ldots$ , and  $R_{M-1}$ , where  $k \times W \le M$ and  $(k+1) \times W > M$ .

**Example 1.** For a  $4 \times 3$  grid as shown in Fig. 1, the set *R* of generalized grid quorums for 2-mutual exclusion is as follows:

$$R = \left\{ \{1, 2, 4, 5\}, \{1, 2, 4, 6\}, \{1, 2, 5, 6\}, \{1, 2, 7, 8\}, \\ \{1, 2, 7, 9\}, \{1, 2, 8, 9\}, \{1, 2, 10, 11\}, \{1, 2, 10, 12\}, \\ \{1, 2, 11, 12\}, \{1, 3, 4, 5\}, \{1, 3, 4, 6\}, \{1, 3, 5, 6\}, \\ \{1, 3, 7, 8\}, \{1, 3, 7, 9\}, \{1, 3, 8, 9\}, \{1, 3, 10, 11\}, \\ \{1, 3, 10, 12\}, \{1, 3, 11, 12\}, \{2, 3, 4, 5\}, \{2, 3, 4, 6\}, \\ \{2, 3, 5, 6\}, \{2, 3, 7, 8\}, \{2, 3, 7, 9\}, \{2, 3, 8, 9\}, \\ \{2, 3, 10, 11\}, \{2, 3, 10, 12\}, \{2, 3, 11, 12\}, \{4, 5, 7, 8\}, \\ \{4, 5, 7, 9\}, \{4, 5, 8, 9\}, \{4, 5, 10, 11\}, \{4, 5, 10, 12\}, \\ \{4, 5, 11, 12\}, \{4, 6, 7, 8\}, \{4, 6, 7, 9\}, \{4, 6, 8, 9\}, \\ \{4, 6, 10, 11\}, \{4, 6, 10, 12\}, \{4, 6, 11, 12\}, \{5, 6, 7, 8\}, \\ \{5, 6, 7, 9\}, \{5, 6, 8, 9\}, \{5, 6, 10, 11\}, \\ \{5, 6, 10, 12\}, \{7, 8, 11, 12\}, \{7, 9, 10, 11\}, \\ \{7, 9, 10, 12\}, \{7, 9, 11, 12\}, \{8, 9, 10, 11\}, \\ \{8, 9, 10, 12\}, \{8, 9, 11, 12\} \right\}.$$

Totally, *R* contains 54 (=  $6 * 3^2$ , where  $6 = C(4, \lceil (4 + 1)/(2+1) \rceil)$ ,  $3 = C(3, \lceil (3+1)/2 \rceil)$ ,  $2 = \lceil (4+1)/(2+1) \rceil$  and

$$C(j,i) = \frac{j \times (j-1) \times \dots \times (j-i+1)}{1 \times 2 \times \dots \times i})$$

quorums. Note that, in this example, every  $R_i$  contains any two nodes in row i,  $0 \le i \le 3$ ; a generalized grid quorum contains any  $\lceil (4 + 1)/(2 + 1) \rceil = 2$ (= W) quorums from  $R_0$ ,  $R_1$ ,  $R_2$  and  $R_3$ . Moreover, it satisfies the conditions:  $k \times W = 2 \times 2 \le 4$  (= M)and  $(k + 1) \times W = 3 \times 2 = 6 > 4$  (= M).

## 3.2. Correctness

In this section, we prove that the set of the generalized grid quorums for k-mutual exclusion is a kcoterie. Here, we will refer to such a k-coterie as the generalized grid coterie.

**Lemma 1.** The set of the k-majority quorums is a k-coterie [6,8].

**Lemma 2.** The set of the majority quorums is a 1-coterie [6].

**Lemma 3.** Let  $U_1$  and  $U_2$  be two nonempty sets of nodes such that  $U_1 \cap U_2 = \emptyset$ , and  $x \in U_1$ . Let  $U = (U_1 - x) \cup U_2$ . The coterie join operation  $\otimes_x$  [7] is defined as

$$\overline{Z} = \overline{X} \otimes_x \overline{Y} = \left\{ CT_x(X, Y) \mid X \in \overline{X}, \ Y \in \overline{Y} \right\},\$$

where  $\overline{X}$  is a k-coterie under  $U_1$ ,  $\overline{Y}$  is a 1-coterie under  $U_2$ , and

$$CT_{x}(X,Y) = \begin{cases} (X - \{x\}) \cup Y & \text{if } x \in X, \\ X & \text{otherwise.} \end{cases}$$

Then,  $\overline{Z}$  is a k-coterie under U [4].

**Theorem 1.** The set of the generalized grid quorums for k-mutual exclusion is a k-coterie.

**Proof.** Let  $row_i$  be the set of nodes in row *i* and  $R_i$  be any one of the majority quorums under  $row_i$ ,  $0 \le i \le (M-1)$ . (For the example shown in Fig. 1,  $R_0$  can be  $\{1, 2\}$ , or  $\{1, 3\}$ , or  $\{2, 3\}$ .) A generalized grid quorum Q contains any  $\lceil (M+1)/(k+1) \rceil$  quorums from  $R_0, R_1, \ldots, R_{M-1}$ . Let  $U_1 = \{g_0, g_1, \ldots, g_{M-1}\}$  and  $U_1 \cap row_i = \emptyset$ ,  $0 \le i \le (M-1)$ . (Note that here, we consider the whole  $row_i$  as a new element  $g_i$ , for example,  $g_0 = \{1, 2, 3\}$  in Fig. 1; therefore,  $\{g_0\} \cap row_0 = \{\{1, 2, 3\}\} \cap \{1, 2, 3\} = \emptyset$ .) Let  $\overline{X}$  be the set of the *k*-majority quorums under  $U_1$ , the quorum

size of  $\overline{X}$  is  $\lceil (M + 1)/(k + 1) \rceil$ . (For the example shown in Fig. 1,  $\{g_0, g_1\}$  is one of the elements in  $\overline{X}$  for k = 2.) Moreover, let  $x = g_0 \in U_1$ ,  $U_2 = row_0$ , and  $\overline{Y}$  be the set of the majority quorums under  $row_0$ . (For the example shown in Fig. 1,  $\overline{Y} = \{\{1, 2\}, \{1, 3\}, \{2, 3\}\}$ .)

Based on Lemma 3, we have that  $\overline{Z}$  is a k-coterie under  $U = (U_1 - \{g_0\}) \cup row_0$ , since  $\overline{X}$  is a k-coterie based on Lemma 1, and  $\overline{Y}$  is a 1-coterie based on Lemma 2. (For the example shown in Fig. 1,  $\overline{Z}$  is a 2coterie under  $U = \{g_0, g_1, g_2, g_3\} - \{g_0\} \cup \{1, 2, 3\} =$  $\{1, 2, 3, g_1, g_2, g_3\}$ , and one of the elements in  $\overline{Z}$ is  $\{g_0, g_1\} - \{g_0\} \cup \{1, 2\} = \{1, 2, g_1\}$ .) Therefore,  $\forall (Q_x \in \overline{X} \text{ under } U_1 \text{ and } g_0 \in Q_x), \text{ we have a new }$ quorum  $(R_0 \cup (Q_x - \{g_0\})) \in \overline{Z}$  under  $(U_1 - \{g_0\}) \cup$ row<sub>0</sub>. That is, a quorum in  $\overline{Z}$  under  $(U_1 - \{g_0\}) \cup row_0$ , contains any  $\lceil (M+1)/(k+1) \rceil$  subsets from  $R_0$ ,  $g_1, \ldots, and g_{M-1}$ . In the same way, we can replace  $U_2$ with every  $row_i$ , where  $1 \le i \le (M - 1)$ . Therefore, a quorum in  $\overline{Z}$  under  $row_0 \cup row_1 \cup \cdots \cup row_{M-1}$  $(=((U_1 - \{g_0, g_1, \dots, g_{M-1}\}) \cup row_0 \cup row_1 \cup \dots \cup$  $row_{M-1}$ ), contains any  $\lceil (M+1)/(k+1) \rceil$  subsets from  $R_0, R_1, \ldots$ , and  $R_{M-1}$ . Consequently, the set of generalized grid quorums for k-mutual exclusion is a *k*-coterie.  $\Box$ 

Note that from this proof, we can show that, in fact, the proposed strategy can be considered as a special case of a hybrid strategy based on the *k*-majority and the majority strategies. That is, for a system with  $M \times N$  nodes, we first divide them into M sets with each set N nodes. Inside each set i of M sets, we apply the majority quorums strategy for 1-mutual exclusion for those N nodes, which results in  $R_i$ , where  $0 \le i \le (M - 1)$ . Between those M sets, we apply the *k*majority strategy.

## 3.3. Availability of the generalized grid quorums

In this section, we first analyze the availability of the majority quorums for 1-mutual exclusion [6] and then the generalized grid quorums for k-mutual exclusion. Here, we assume that all the nodes have the same upprobability p, which is the probability that a single node is up operational.

For the availability of the majority strategy, let AVM(j) be the function evaluating the probability that the majority quorums can be formed with j nodes

simultaneously, and  $C(j, i) = \frac{j \times (j-1) \times \dots \times (j-i+1)}{1 \times 2 \times \dots \times i}$ . Function AVM(j) has the following condition:

$$AVM(j) = \sum_{i=\lceil (j+1)/2\rceil}^{j} C(j,i) \times p^{i} \times (1-p)^{j-i}.$$

Next, for the availability of the generalized grid quorum strategy, let (k, l)-availability,  $1 \le l \le k$ , be the probability that l pairwise disjoint quorums of a k-coterie can be formed successfully; it is used as a measure for the fault-tolerant ability of a solution using a k-coterie.

Let AVG(l) be the function evaluating the probability that l pairwise disjoint quorums under grid can be formed simultaneously. The function AVG(l) has the following two boundary conditions:

$$AVM(j) = \sum_{i=\lceil (j+1)/2\rceil}^{J} C(j,i) \times p^{i} \times (1-p)^{j-i}.$$
 (1)

$$AVG(l) = \sum_{i=l \times \lceil (M+1)/(k+1)\rceil}^{M} C(M,i) \times AVM(N)^{i} \times \left(1 - AVM(N)\right)^{M-i}. (2)$$

## 4. A comparison

In this section, we make a comparison of the generalized grid quorum, k-majority, cohorts, and DIV quorum strategies in terms of the quorum size and the availability, where we assume that the system has a fully connected network topology and no communication failure will occur. However, a node failure can occur. (Note that, here, we assume that a failed node simply stops execution (i.e., a fail-stop system). That is, no Byzantine failure occurs.)

The number of messages required to construct a quorum is proportional to the quorum size. The quorum size of the generalized grid quorum strategy is equal to  $\lceil (M + 1)/(k + 1) \rceil \times \lceil (N + 1)/2 \rceil$ , where *M* is the number of row and *N* is the number of columns in a grid. Note that, in the generalized grid quorum strategy, *total* nodes are divided into *M* rows. Between rows, we apply the *k*-majority strategy, and inside each row, we apply the majority strategy. Therefore, in each row, the quorum size is equal to  $\lceil (N + 1)/2 \rceil$ , and the

Case	total	DIV	k-majority	G-Grid	Cohorts l
	$(= M \times N)$	$\lceil \frac{total+k}{2k} \rceil$	$\lceil \frac{total+1}{k+1} \rceil$	$\lceil \frac{M+1}{k+1} \rceil \times \lceil \frac{N+1}{2} \rceil$	$(total = \sum_{i=1}^{l}  C_i )$
1	$81$ $(=9 \times 9)$	11	17	10(*)	12(**) (= 4 + 7 × 11)
2	95 (= 19 × 5)	13	20	12(*)	14(**) (= 4 + 7 × 13)
3	99 (=9×11)	13	20(*)	12(*)	$14$ $(=4+7\times9+8\times4)$
4	$154 \\ (= 14 \times 11)$	20	31(*)	18(*)	22 $(=4+7 \times 18+8 \times 3)$
5	132 (= 4 × 33)	17(*)	27	17(*)	$   19    (= 4 + 7 \times 16 + 8 \times 2) $
6	$252 \\ (= 4 \times 63)$	32(*)	51	32(*)	37(**) (= 4 + 7 × 36)
7	$44 \\ (= 4 \times 11)$	6(*)	9(*)	6(*)	$ \begin{array}{c} 6\\ (=4+8\times5) \end{array} $
8	$ \begin{array}{c} 84 \\ (=4 \times 21) \end{array} $	11(*)	17(*)	11(*)	$12$ $(=4+7\times8+8\times3)$
5A	$132 (= 44 \times 3)$	17(*)	27	18(*)	19
6A	$252 \\ (= 84 \times 3)$	32(*)	51	34(*)	37(**)
1A	81 (= 27 × 3)	11	17	12	12(**)
3A	99 (= 33 × 3)	13	20(*)	14	14
5B	$132 (= 12 \times 11)$	17(*)	27	18	20

Table 1 A comparison of the quorum size: k = 4

(\*) the case of  $\lceil x/y \rceil = x/y$ .

(\*\*) the case of l = (total - k)/(2k - 1) + 1.

quorum size of the generalized grid quorum strategy is equal to  $\lceil (M+1)/(k+1) \rceil \times \lceil (N+1)/2 \rceil$ .

The quorum size of the *k*-majority strategy is equal to  $\lceil (total + 1)/(k + 1) \rceil$  [6,8]. For example, when there are 1, 2, 3, 4 nodes in the system, the set *R* of 2-majority quorums is as follows: *R* = {{1, 2}, {1, 3}, {1, 4}, {2, 3}, {2, 4}, {3, 4}}. The quorum

size of the DIV strategy is equal to  $\lceil (total+k)/2k \rceil$  [1]. For example, when there are 1, 2, 3, 4, 5, 6 nodes in the system and we divided nodes into two classes, (1, 2, 3) and (4, 5, 6), the set *R* of DIV of majority quorums for 2-mutual exclusion is as follows:  $R = \{1, 2\}, \{1, 3\}, \{2, 3\}, \{4, 5\}, \{4, 6\}, \{5, 6\}\}$ . A cohort structure Coh(k, l) has *l* pairwise disjoint co-

Case	total	DIV	k-majority	G-Grid	Cohorts l
	$(= M \times N)$	$\lceil \frac{total+k}{2k} \rceil$	$\lceil \frac{total+1}{k+1} \rceil$	$\lceil \frac{M+1}{k+1} \rceil \times \lceil \frac{N+1}{2} \rceil$	$(total = \sum_{i=1}^{l}  C_i )$
1	133	23	34	20(*)	27(**)
	$(=19 \times 7)$				$(=3+5 \times 26)$
	$(=7 \times 19)$				
2	203	35	51	30(*)	41(**)
	$(=7 \times 29)$				$(=3+5\times40)$
3	35	7	9(*)	6(*)	7
	$(=7 \times 5)$				$(=3+5\times4+6\times2)$
4	143	25	36(*)	21(*)	29(**)
	$(= 11 \times 13)$				$(=3+5 \times 28)$
5	63	11(*)	16(*)	10(*)	13(**)
	$(=7 \times 9)$				$(=3+5 \times 12)$
6	45	8(*)	12	8(*)	9
	$(= 3 \times 15)$				$(=3+5\times 6+6\times 2)$
	$(=15 \times 3)$				
7	93	16(*)	24	16(*)	19(**)
	$(=3 \times 31)$				$(=3+5 \times 18)$
	$(=31 \times 3)$				
8	15	3(*)	4(*)	3(*)	3
	$(=3 \times 5)$				$(=3+6 \times 2)$
9	123	21(*)	31(*)	21(*)	25(**)
	$(= 3 \times 41)$				$(=3+5 \times 24)$
5A	63	11(*)	16(*)	12	13(**)
	$(=9 \times 7)$				
6A	45	8(*)	12	9	9
	$(=9 \times 5)$				

Table 2
A comparison of the quorum size: $k = 3$

(\*) the case of  $\lceil x/y \rceil = x/y$ .

(\*\*) the case of l = (total - k)/(2k - 1) + 1.

horts with the first cohort having *k* members and the other cohorts having more than (2k - 2) members. The quorum size of the cohorts strategy varies from 2 (when k = 1) or *k* (when k > 1) to l = (n - k)/s + 1, for a cohort structure  $Coh(k, l) = \langle k, s, ..., s \rangle$ ,  $l \gg s$  [5]. In fact, the upper bound of the quorum size of the cohorts strategy depends on the structure of cohorts. For example, the following sets are quorums under  $Coh(2, 2) = (\{1, 2\}, \{3, 4, 5\}): Q_1 = \{3, 4\}, Q_2 =$ 

 $\{3, 5\}, Q_3 = \{4, 5\}, Q_4 = \{1, 3\}, Q_5 = \{1, 4\}, Q_6 = \{1, 5\}, Q_7 = \{2, 3\}, Q_8 = \{2, 4\}, \text{ and } Q_9 = \{2, 5\} [5].$ 

Since the quorum size of the cohorts strategy is variable depending on the given cohort structure, in this comparison, we consider the cohort structure as (1)  $\langle k, 2k - 1, ..., 2k - 1 \rangle$ , when  $(total - k) \mod (2k - 1) = 0$ , or (2)  $\langle k, 2k - 1, ..., 2k - 1, 2k, ..., 2k \rangle$ , when  $(total - k) \mod (2k - 1) \neq 0$ . In this case, the range of the quorum size of the cohort coterie for Coh(k, l) Y.-I. Chang, B.-H. Chen / Information Processing Letters 80 (2001) 205-212

changes from k (= (2k - 1) - (k - 1)) to l. Tables 1 and 2 show a comparison of quorum size of those strategies for k = 4 and 3, respectively, where the generalized grid quorum strategy is denoted as G-Grid. Note that the case marked with (\*) in the DIV, k-majority and G-Grid strategies means  $\lceil x/y \rceil = x/y$ , i.e.,  $x \mod y = 0$ . While the mark (\*\*) attached with l in the cohorts strategy denotes the case of  $(total - k) \mod (2k - 1) = 0$ .

In Table 1, when k = 4, for cases 1–4, the quorum size of the G-Grid strategy is smaller than that of the DIV and *k*-majority strategies. For cases 5–8, the quorum size of the G-Grid strategy is the same as that of the DIV strategy, and is smaller than that of the *k*-majority strategy. For cases 1–8, the quorum size of the G-Grid strategy is smaller than or equal to the level (= *l*) of the cohorts strategy. For the other cases (5A, 6A, 1A, 3A and 5B), the quorum size of the G-Grid strategy is larger than that of the DIV strategy, smaller than or equal to the level (= *l*) of the cohorts strategy, and is smaller than or equal to the level (= *l*) of the cohorts strategy, and is smaller than that of the *k*-majority strategy, and is smaller than that of the *k*-majority strategy, and is smaller than or equal to the level (= *l*) of the cohorts strategy. Note that for cases 5 and 5A, both satisfy ( $\lceil x/y \rceil = x/y$ ) in the G-Grid strategy, but they provide different quorum size.

Similarly, in Table 2, when k = 3, for cases 1–5, the quorum size of the G-Grid strategy is smaller than that of the DIV and *k*-majority strategies. For cases 6–9, the quorum size of the G-Grid strategy is the same as that of the DIV strategy, and is smaller than that of the *k*-majority strategy. For cases 1–9, the quorum size of the G-Grid strategy is smaller than or equal to the level (= *l*) of the cohorts strategy. For the other cases (5A and 6A), the quorum size of the G-Grid strategy is larger than that of the DIV strategy, and is smaller than or equal to the level (= *l*) of the cohorts strategy, and is smaller than that of the *k*-majority strategy, and is smaller than or equal to the level (= *l*) of the cohorts strategy. Note that for case 5, both the G-Grid and DIV strategies satisfy ( $\lceil x/y \rceil = x/y$ ), but the quorum size of the DIV strategy.

In summary, for the DIV and G-Grid strategies, the quorum size of the G-Grid strategy can be smaller than (case 5 for k = 3), equal to (case 5 for k = 4) or larger than that of the DIV strategy (case 5A for k = 4). For the *k*-majority and G-Grid strategies, the quorum size of the G-Grid strategy is smaller than (case 3 for k = 4) that of the *k*-majority strategy. Moreover, when some node failures occur, the quorum size of the G-Grid, DIV and *k*-majority strategies are still fixed, and

Table 3				
A comparison of the availability:	$total = 4 \times$	33 =	132, k	= 4

l	р	The availability	
1	$0$	k-majority > cohorts > G-Grid = DIV	
	$0.55$	k-majority > G-Grid = DIV > cohorts	
	$0.65$	G- $G$ rid = $k$ -majority = $DIV > cohorts$	
2	$0$	cohorts > $k$ -majority > G-Grid = DIV	
	$0.3$	k-majority > cohorts > G-Grid = DIV	
	$0.5$	k-majority > G-Grid = DIV > cohorts	
	$0.7$	G- $G$ rid = $DIV = k$ -majority > cohort	
3	$0$	cohort > G-Grid = DIV > k-majority	
	$0.4$	G-Grid = DIV > cohorts > k-majority	
	$0.65$	G- $Grid = DIV > k$ -majority > cohorts	
	$0.7$	G- $G$ rid = $DIV = k$ -majority > cohorts	
4	$0$	<i>k</i> -majority > cohorts > G-Grid = DIV	
	$0.2$	cohorts > G-Grid = DIV > k-majority	
	$0.3$	G-Grid = DIV > cohorts > k-majority	
	$0.7$	G-Grid = DIV = $k$ -majority > cohorts	

Table 4

A comparison of the availability:  $total = 19 \times 7 = 133$ , k = 3

l	р	The availability
1	$0$	cohorts > $k$ -majority > G-Grid > DIV
	$0.2$	k-majority > cohorts > G-Grid > DIV
	$0.4$	k-majority > G-Grid > cohorts > DIV
	$0.5$	k-majority > G-Grid > DIV > cohorts
	$0.6$	G-Grid = $k$ -majority > DIV > cohorts
	$0.65$	G-Grid = $k$ -majority = DIV > cohorts
2	$0$	cohorts > G-Grid > DIV > k-majority
	$0.45$	G-Grid = DIV > cohorts > k-majority
	$0.5$	DIV > <i>k</i> -majority > G-Grid > cohorts
	$0.55$	<i>k</i> -majority > DIV > G-Grid > cohorts
	$0.7$	G-Grid = DIV = $k$ -majority > cohorts
3	$0$	cohorts > DIV > G-Grid > k-majority
	$0.4$	DIV > cohorts > G-Grid > k-majority
	$0.5$	DIV > G-Grid > cohorts > k-majority
	$0.7$	DIV > G- $Grid > k$ -majority > cohorts
	$0.8$	G- $G$ rid > $DIV > k$ -majority > cohorts

these three strategies can be always fault-tolerant up to  $(total - k \times (the number of a quorum size))$  node failures. While in the cohorts strategy, the quorum size can be increased as the number of failure nodes is increased.

Tables 3 and 4 show a comparison of the availability. From this table, we show that the availability of the G-Grid strategy can be better than (or equal to) that of other strategies when p near 1 (i.e., low probability of node failures). From our several more simulation results, we observe that for a given total, (1) when the quorum size of strategy A is smaller than that of strategy  $B \ (\neq A)$ , it does not imply that the availability of strategy A will definitely be higher than that of strategy B; (2) when strategies A and B have the same quorum size, the availability of the strategies can be the same (case 5 for k = 4) or different (case 7 for k = 3, total =  $31 \times 3$ ; (c) for the same G-Grid strategy, when the quorum size is the same for different  $M \times N$ , the availability for different  $M \times N$  can be different (case 7 for k = 3).

## 5. Conclusion

In this paper, we have proposed a strategy called generalized grid quorum for k-mutual exclusion, which imposes a logical grid structure on the network. In general, in the generalized grid quorum strategy, *total* nodes are divided into M rows. Between rows, we have applied the k-majority strategy, and inside each

row, we have applied the majority strategy. Therefore, the proposed strategy can be considered as a hybrid approach which contains the k-majority and the majority strategies. From our performance study, we have shown that the generalized grid quorum strategy can provide a good performance in terms of the quorum size and the availability. How to extend the generalized grid quorum strategy to tolerate even more node failures is the future research direction.

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